Regional Height–Diameter Equations for Major Tree Species of Southwest Oregon

Hailemariam Temesgen, David W. Hann, and Vincente J. Monleon

ABSTRACT

Selected tree height and diameter functions were evaluated for their predictive abilities for major tree species of southwest Oregon. Two sets of equations were evaluated. The first set included four base equations for estimating height as a function of individual tree diameter, and the remaining 16 equations enhanced the four base equations with alternative measures of stand density and relative position. The inclusion of the crown competition factor in larger trees (CCFL) and basal area (BA), which simultaneously indicates the relative position of a tree and stand density, into the base height–diameter equations increased the accuracy of prediction for all species. On the average, root mean square error values were reduced by 45 cm (15% improvement). On the basis of the residual plots and fit statistics, two equations are recommended for estimating tree heights for major tree species in southwest Oregon. The equation coefficients are documented for future use.

Keywords: basal area in larger trees index, southwest Oregon

Accurate growth and yield estimates are essential for sustainable forest management. Modeling stand development over time relies on accurate estimates of tree height (*H*) and diameter (*D*). Accurate *H* measures are required for describing vertical stand structure (e.g., Dubrasich et al. 1997) and for estimating stand volume and site quality. However, measuring *H* is costly and, as a result, trees are often subsampled for *H*, with the subsample often concentrated in the trees of greatest diameter, for example those used to estimate site index. Subsampled trees can also be used to localize regional height–diameter (*H-D*) functions (e.g., Wykoff et al. 1982, Hann 2005).

Many growth and yield models require *H* and *D* as basic input variables, with all or part of the *H*s predicted from measured *D*s using *H-D* functions (Burkhart et al. 1972, Wykoff et al. 1982, Huang et al. 1992, Hann 2005). *H-D* functions can also be used to indirectly predict height growth (Larsen and Hann 1987). For example, in the southwest Oregon version of the ORGANON (SWO-ORGANON) growth-and-yield model (Hann 2005), missing heights are directly predicted using the species-specific *H-D* equations of Hanus et al. (1999a), and these equations are also used to estimate height growth for minor species

Conventionally, *H-D* models have used only *D* as a predictor variable. However, several studies have explored the use of additional variables that might influence the *H-D* relationship both within and between stands. For example, stand density measures have been used by Larsen and Hann (1987), Staudhammer and LeMay (2000), and Temesgen and Gadow (2004); relative tree position variables have been used by Temesgen and Gadow (2004); site quality variables have been by Larsen and Hann (1987) and Wang and Hann (1988); and the average *H* and *D* of the top height trees have been used by Krumland and Wensel (1978) and Hanus et al. (1999b) for even-aged stands. As expected, inclusion of these variables improved *H* estimates.

The objective of this article is to assess the predictive abilities of four commonly used *H-D* equations for major tree species of southwest Oregon. The study also evaluates the contribution of relative position of trees and stand density variables in estimating *H-D* relationships.

Methods

Data

The data were collected in two studies associated with the development of SWO-ORGANON. The first set of data was collected during 1981, 1982, and 1983 as part of the southwest Oregon Forestry Intensified Research (FIR) Growth and Yield Project. This study included 391 plots in an area extending from near the California border (42°10′ north) in the south, to Cow creek (43°00′ north) in the north, and from the Cascade crest (122°15′ west) on the east to approximately 15 miles west of Glendale (123°50′ west). Elevation of the sample plots ranged from 250 to 1,600 meters. Selection was limited to stands under 120 years of age and 80% basal area in conifer species. The second study covered approximately the same area but extended the selection criterion to include stands with trees over 250 years in age and to younger stands with a greater component of hardwoods. An additional 138 plots were measured in this second study. Stands treated in the previous 5 years were not sampled in either study.

A total of 30 tree species were found on these 529 plots. The most common coniferous species was Douglas-fir [*Pseudotsuga menziesii* (Beissn.) Franco] (found on 339 plots), followed by incense-cedar [*Calocedrus decurrens* (Torr.) Florin] (110 plots), grand fir [*Abies grandis* (Dougl. ex D. Don) Lindl.] (115 plots), ponderosa pine

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H. Temesgen (hailemariam.temesgen@oregonstate.edu) is assistant professor, Department of Forest Resources, Oregon State University, 237 Peavy Hall, Corvallis, OR 97331-5703. David W. Hann (david.hann@oregonstate.edu) is professor, Department of Forest Resources, Oregon State University, 237 Peavy Hall, Corvallis, OR 97331-5703. Vincente J. Monleon (vjmonleon@fs.fed.us) is research mathematical statistician, Forest Inventory and Analysis, PNW Research Station, 620 SW Main St., Suite 400, Portland, OR 97205. We thank Aaron Weiskittel for his insights and comments.

Table 1. Number of plots and trees and minimum (min), average (mean), maximum (max), and SD of the dbh, height, basal area in larger trees, and crown competition factor in larger trees by species.

			Diameter (cm)				Height (m)				Basal area in larger trees (m^2/ha)				CCFL			
Species	Plots (*n*)	Trees (*n*)	Min	Mean	Max	SD	Min	Mean	Max	SD	Min	Mean	Max	SD	Min	Mean	Max	SD
Incense-cedar	110	623	0.2541	22.84	174.82	24.51	1.40	11.87	55.99	10.44	0.00	27.09	92.81	19.73	0.00	121.36	485.61	94.90
White/grand fir	115	907	0.2541	32.91	113.84	21.64	1.40	23.50	61.23	12.66	0.46	30.22	81.51	18.78	1.53	101.48	349.52	61.46
Sugar pine	56	124	1.2705	52.34	135.44	25.90	1.95	27.76	47.52	10.43	0.00	17.22	63.72	15.61	0.00	55.08	342.65	59.59
Ponderosa pine	81	481	0.5082	36.78	112.57	21.88	1.49	24.01	53.86	12.36	0.00	18.90	64.29	14.67	0.00	66.35	418.73	59.45
Douglas-fir	339	7,953	0.2541	35.94	206.58	24.54	1.40	25.13	74.43	12.83	0.00	24.26	93.37	16.14	0.00	103.18	485.61	74.86
Pacific madrone	148	774	0.2541	19.92	78.52	10.67	1.43	13.42	30.39	5.82	0.00	27.33	84.49	15.40	0.00	125.93	378.50	67.33
Golden chinkapin	85	460	0.2541	11.56	53.62	9.77	1.52	8.19	26.37	5.75	0.00	25.72	72.58	18.23	0.00	143.41	465.70	76.86
Tanoak	44	215	0.2541	8.79	53.87	10.30	1.40	6.88	30.69	5.91	1.42	37.48	87.09	19.11	9.89	226.41	461.32	88.46
Canyon live oak	42	151	0.2541	8.84	38.62	6.41	1.43	6.13	17.65	3.34	5.84	37.77	83.60	17.16	62.22	186.67	317.08	56.40
California black oak	32	140	0.5082	28.39	87.92	21.71	1.62	14.04	33.86	6.06	0.00	28.71	61.33	16.33	0.00	139.31	341.41	89.93

(*Pinus ponderosa* Dougl. ex Laws.) (81 plots), sugar pine (*Pinus lamertiana* Dougl.) (56 plots), and white fir [*Abies concolor* (Gord. & Glend.) Lindl. ex Hildebr.] (115 plots). The most common hardwood species was Pacific madrone (*Arbutus menziesii* Pursh) (found on 81 plots), followed by golden chinkapin [*Castanopsis chrysophylla* (Dougl.) A. DC.] (85 plots), California black oak (*Quercus kelloggi* Newb.) (32 plots), canyon live oak (*Quercus chrysolepis* Liebm.) (42 plots), and tanoak [*Lithocarpus densiflorus* (Hook & Arn.) Rehd.] (44 plots). The number of species found on a single plot ranged from 1 to 12 and averaged almost five species.

Stand structures found in the sample area range from even-aged stands of one or two stories to uneven-aged stands. Of the 529 stands sampled, 363 were classified as having an even-aged overstory and 166 were classified as uneven-aged in structure.

In both studies, each stand was sampled with 4–25 sample points spaced 45.73 m apart. The sampling grid was established in a manner such that all sample points were at least 30.5 m from the edge of the stand. At each sample point, trees were sampled with a nested plot design composed of four subplots: trees 10.2 cm or less in dbh (D) were selected on a circular subplot with a fixed 2.37-m radius, trees 10.3–20.3 cm D were selected on a circular subplot with a fixed 4.74-m radius, trees 20.4 to 91.4-cm D were selected on a 4.592-basal area factor (BAF) variable radius subplot, and trees over 91.4-cm D were selected on a 13.776-BAF variable radius subplot.

Measurements of H and D were taken on all sample trees. D was measured to the nearest tenth of an inch, rounded down, with a diameter tape. H was measured on all trees either directly with a 7.6–13.7-m telescoping fiber glass pole or, for taller trees, indirectly using the pole-tangent method (Larsen and Hann 1987) and recorded to the nearest 0.03 m. For trees with broken or dead tops, H was measured to the top of the live crown. All trees were assessed for type and severity of damage, and time since last cutting was determined on all previously treated plots.

From each untreated plot, heights and diameters of undamaged trees were extracted to assess and evaluate selected height–diameter functions and to compare selected equation forms. To obtain stable parameter estimates, only species with at least 100 sample trees spread over at least 20 plots in each data set were included in this study. This restriction resulted in 10 tree species being available for equation evaluation. Following Larsen and Hann (1987) and Hanus et al. (1999a), two of these species, white fir and grand fir, were combined because of their tendency to interbreed in southwest Oregon. Summaries of the data sets are found in Tables 1 and 2.

Table 2. Minimum, average (mean), maximum, and SD of basal area per hectare (BA) and crown competition factor (CCF) for the equation building and equation evaluation data sets.

	Minimum	Mean	Maximum	SD
BA (m^2/ha)	0.32	47.02	101.02	19.33
CCF	26.63	249.74	536.75	82.47
SI (m)	12.65	30.72	44.78	5.35

SI, site index.

Data Analysis

The relative predictive ability of different model forms, as well as the estimated parameters, varies from species to species (Larsen and Hann 1987, Garman et al. 1995, Temesgen and LeMay 1999). Based on a literature review, five sets of four equations (for a total of 20 alternative equations) were evaluated. The first set included four base equations for estimating height as a function of individual tree diameter alone, and the remaining four sets of equations estimated height as a function of individual tree diameter and various tree- and stand-level variables. Because the height–diameter functions are expected to vary by tree species, the selected equations were fitted and evaluated for accuracy by species.

Base Height–Diameter Equations

We chose to evaluate four alternative height–diameter equations that have been previously used for tree species in the Pacific Northwest. The following Weibull-based equation was applied by Yang et al. (1978) to tree species in British Columbia:

$$H = 1.3 + a[1 - e^{bD^c}] + \varepsilon_1, \tag{1}$$

where a, b, and c are species-dependent coefficients; $a > 0$; $b < 0.0$; e is the Naperian constant (i.e., 2.718); a represents asymptotic height; and b and c determine the rate of increase. Huang et al. (1992) found Equation 1 to be consistently the best among the 19 height–diameter functions they tested.

Garman et al. (1995) used the following Chapman–Richards function (Richards 1959) for 24 western Oregon tree species:

$$H = 1.3 + a[1 - e^{bD}]^c + \varepsilon_2, \tag{2}$$

where a controls the asymptotic height; b is the steepness parameter, which is always negative; and c is the curvature parameter. Equations 1 and 2 are concave downward without an inflection point.

Table 3. List of equations examined in this study.[a]

Equation	Form
1: Yang et al. (1978)	$\hat{H} = 1.3 + a[1 - e^{bD^c}]$
2: Chapman-Richards (1959) and Garman et al. (1995).	$\hat{H} = 1.3 + a[1 - e^{bD}]^c$
3: Ratkowsky (1990)	$\hat{H} = 1.3 + e^{[a + b/(D + c)]}$
4: Hanus et al. (1999)	$\hat{H} = 1.3 + e^{[a + bD^c]}$
5: Equation 1 with BAL and BA	$\hat{H} = 1.3 + a[1 - e^{bD^c}]$, where $a = a_0 + a_1 \times \mathrm{BAL} + a_2 \times \mathrm{BA}$
6: Equation 2 with BAL and BA	$\hat{H} = 1.3 + a[1 - e^{bD}]^c$, where $a = a_1 + a_2 \times \mathrm{BAL} + a_2 \times \mathrm{BA}$
7: Equation 3 with BAL and BA	$\hat{H} = 1.3 + e^{[a + b/D + c]}$, where $a = a_0 + a_1 \times \mathrm{BAL} + a_2 \times \mathrm{BA}$
8: Equation 4 with BAL and BA	$\hat{H} = 1.3 + e^{[a + b \times D^c]}$, where $a = a_0 + a_1 \times \mathrm{BAL} + a_2 \times \mathrm{BA}$
9: Equation 1 with CCFL and CCF	$\hat{H} = 1.3 + a[1 - e^{bD^c}]$, where $a = a_0 + a_1 \times \mathrm{CCFL} + a_2 \times \mathrm{CCF}$
10: Equation 2 with CCFL and CCF	$\hat{H} = 1.3 + a[1 - e^{bD}]^c$, where $a = a_1 + a_2 \times \mathrm{CCFL} + \mathrm{a}_3\,\mathrm{CCF}$
11: Equation 3 with CCFL and CCF	$\hat{H} = 1.3 + e^{[a + b/D + c]}$, where $a = a_0 + a_1 \times \mathrm{CCFL} + a_2 \times \mathrm{CCF}$
12: Equation 4 with CCFL and CCF	$\hat{H} = 1.3 + e^{[a + b \times D^c]}$, where $a = a_0 + a_1 \times \mathrm{CCFL} + a_2 \times \mathrm{CCF}$
13: Equation 1 with CCFL and BA	$\hat{H} = 1.3 + a[1 - e^{bDc}]$, where $a = a_0 + a_1 \times \mathrm{CCFL} + a_2 \times \mathrm{BA}$
14: Equation 2 with CCFL and BA	$\hat{H} = 1.3 + a[1 - e^{bD}]^c$, where $a = a_1 + a_2 \times \mathrm{CCFL} + a_3 \times \mathrm{BA}$
15: Equation 3 with CCFL and BA	$\hat{H} = 1.3 + e^{[a + b/(D + c)]}$, where $a = a_0 + a_1 \times \mathrm{CCFL} + \mathrm{a}_2 \times \mathrm{BA}$
16: Equation 4 with CCFL and BA	$\hat{H} = 1.3 + e^{[a + b \times D^c]}$, where $a = a_0 + a_1 \times \mathrm{CCFL} + a_2 \times \mathrm{BA}$
17: Equation 1 with BAL and CCF	$\hat{H} = 1.3 + a[1 - e^{bDc}]$, where $a = a_0 + a_1 \times \mathrm{BAL} + a_2 \times \mathrm{CCF}$
18: Equation 2 with BAL and CCF	$\hat{H} = 1.3 + a[1 - e^{bD}]^c$, where $a = a_1 + a_2 \times \mathrm{BAL} + a_3 \times \mathrm{CCF}$
19: Equation 3 with BAL and CCF	$\hat{H} = 1.3 + e^{[a + b/(D + c)]}$, where $a = a_0 + a_1 \times \mathrm{BAL} + a_2\,\mathrm{CCF}$
20: Equation 4 with BAL and CCF	$\hat{H} = 1.3 + e^{[a + b + D^c]}$, where $a = a_0 + a_1 \times \mathrm{BAL} + a_2 \times \mathrm{CCF}$

[a] Where *a*, *b*, and *c* are species-dependent coefficients, $\hat{H}$ is estimated tree height in meters; *D* is observed diameter outside bark at breast height in cm; and *e* is the Naperian constant (*i.e.*, 2.718). BAL, basal area in larger trees (m^2/ha); BA, basal area per hectare (m^2/ha); CCF, crown competition factor; CCFL, crown competition in larger trees.

Flewelling and de Jong (1994) used the following function from Ratkowsky (1990) for western hemlock in the coastal region of the Pacific Northwest:

$$H = 1.3 + e^{[a+b/(D+c)]} + \varepsilon_3. \qquad (3)$$

A modified variant of Equation 3 in which *c* is fixed to 1.0 has been used for the 11 tree species found in the Northern Idaho variant of Prognosis (Wykoff et al. 1982). Setting the value of *c* in Equation 3 to 0.0 and allowing the resulting power term on *D* to vary from −1.0 results in the following equation:

$$H = 1.3 + e^{[a+bD^c]} + \varepsilon_4. \qquad (4)$$

In Equations 3 and 4, e^a represents the asymptotic height. Parameters *b* and *c* represent steepness and curvature, respectively. Parameter *c* in Equation 4 should always have a negative sign. For a total of 26 tree species in the Pacific Northwest, Equation 4 has been used by Larsen and Hann (1987), Wang and Hann (1988), Dolph (1989), and Hanus et al. (1999a, 1999b).

Addition of Relative Position and Stand Density Measures to the Base Equations

A tree's crown length affects the form of the tree and, as a result, the *H-D* relationship (Larson 1963). Factors affecting crown length include the relative position of the tree within the stand and the stand's density (Ritchie and Hann 1987, Zumrawi and Hann 1989, Hanus et al. 2000, Hann et al. 2003). It is expected that a decrease in either relative position or an increase in stand density would result in an increase in predicted height. Two stand density measures [basal area per hectare (BA) in m^2/ha and crown competition factor (CCF)], and two relative position measures [CCFL and basal area per hectare in larger trees (BAL) in m^2/ha] were evaluated in this study. All four combinations of pairs of stand density and relative position measures (i.e., BAL and BA, CCFL and CCF, CCFL and BA, and BAL and CCF) were evaluated for potential improvement in the predictive abilities of each of the four base equations. This was accomplished by expressing the asymptotic parameter in each equation (the *a* parameter) as a function of these variables (Table 3). For example, inserting the first set of relative position and stand density measures into Equation 1 can be expressed as follows:

$$H = 1.3 + [a_0 + a_1\mathrm{BAL} + a_2\mathrm{BA}][1 - e^{bD^c}] + \varepsilon_5. \qquad (5)$$

In this equation, the asymptote parameter is assumed to vary linearly with BAL of a subject tree and BA of stand in which the tree resides.

Parameter Estimation

Equation parameters were estimated with the Gauss–Newton optimization technique (Gallant 1987) in a weighted, nonlinear least squares procedure using SAS software (PROC NLIN; SAS Institute, Inc., 2005). A weight of 1.0/DBH was used on the basis of the findings of Larsen and Hann (1987). Initial approximations for each parameter were obtained from linear transformation of the equations, where possible. The starting value of each parameter was varied to find a global minimum, and the run with the smallest MSE was chosen as providing the final parameter estimates. The assumption of homostocedasticity of the weighted residuals was tested using the Goldfield–Quandt test (Goldfield and Quandt 1965). The test indicated homogenous variances over the full range of predicted values at a 0.05 α level.

Equation Comparison and Selection

The fits of the equations were examined using residual plots and a jackknife, exclude-one-stand validation technique (Stone 1974). In the later technique, one stand was excluded from the data set, and the selected models were fitted to the rest of the stands. Then, the models were used to predict the height of all the trees in the excluded stand. The same process was repeated for every stand in the data set. The rationale behind this method is that trees from the same plot tend to be correlated. Excluding a stand from the data set to examine the performance of the models provides stronger model evaluation for a new stand that was not included in the original data set (Monleon et al. 2004).

The following jackknife cross-validation statistic was applied to the cumulative data from the excluded stands to evaluate the per-

Table 4. Bias and root mean square error (RMSE) in meters by species and equation.

			Bias				RMSE			
Species	Plots (*n*)	Trees (*n*)	1	2	3	4	1	2	3	4
Incense-cedar	110	623			0.1		**2.7***	**2.7***	3.0	2.8
White/grand fir	115	907			0.1		4.4	4.4	4.4	**4.3***
Sugar pine	56	124					**4.3***	**4.3***	**4.3***	4.4
Ponderosa pine	81	481					**4.4***	**4.4***	**4.4***	**4.4**
Douglas-fir	339	7,953			0.1		4.2	4.2	4.3	**4.1***
Pacific madrone	148	774					**3.0***	**3.0***	**3.0***	**3.0***
Golden chinkapin	85	460					2.2	2.2	**2.1***	2.2
Tanoak	44	215	0.1	0.1	0.2	0.1	1.9	1.9	2.0	**1.8**
Canyon live oak	42	151					**1.6***	**1.6***	**1.6***	**1.6***
California black oak	32	140	0.1	0.1		0.1	3.7	3.6	**3.5***	**3.5**

Boldface indicates the absolute lowest RMSE value across all equations for a given species. An asterisk (*) indicates the lowest RMSE for unbiased equations only within a species.

formance of the equations:

$$\text{Bias} = \frac{\sum_{i=1}^{n}(H_i - \hat{H}_i)}{n}, \qquad (6)$$

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(H_i - \hat{H}_i)^2}{n}}.$$

where RMSE is the estimate of root mean square error (a measure of accuracy of the prediction); n is the number of trees of a given species; H_i is measured height; and $\hat{H}_i$ is estimated height from the cross-validation study. It should be noted that because RMSE does not remove the effect of bias in its calculation, any bias in the equation will increase its RMSE. Both statistics were rounded to the nearest 0.1 m, which corresponds to the precision one could reasonably expect when measuring H.

The "best" equation was selected in the following fashion:

1. The equation with the lowest RMSE was noted for each species.
2. If the equation with the lowest RMSE was severely biased (i.e., more than 10 cm), then the unbiased equation with the lowest RMSE was also noted for that species.
3. The equations were then evaluated for the consistency of their performance across all of the species.
4. Finally, the performance of the base equations for estimating heights of large trees and trees outside the diameter range of the modeling data was also examined by predicting the heights for the largest five trees of each species and the maximum diameter and height values observed by Garman et al. (1995) and by comparing those predictions with the actual measured heights.

After evaluating the results, one base equation and one expanded equation were selected for estimating height of undamaged trees growing in untreated stands of southwest Oregon. Each equation's parameters were evaluated to determine whether they were significantly different from 0 using the asymptotic t-test. Parameters not significantly different from 0 at $P = 0.05$ were set to 0, and the remaining parameter was re-estimated. Final parameter estimates and their fit statistics are provided for the selected equations.

Results and Discussion

Evaluation of Base Equations

All four base equations provided adequate performance across species (Table 4). Equations 1 and 2 provided the lowest RMSE for five species, whereas Equations 3 and 4 provided the lowest RMSE for 6 and 7 of the 10 species groups, respectively. However, Equations 1, 2, and 4 did prove unbiased for 8 of the 10 species groups, whereas Equation 3 was unbiased for six species groups. In most cases, the biases were within the minimum recognized threshold of ±0.1 m. Considering both bias and RMSE, Equation 3 provided the lowest unbiased RMSE for 6 of the 10 species groups, followed by Equations 1 and 2 with the lowest unbiased RMSE for five species groups, and Equation 4 for five species groups.

Sizable differences were observed in the abilities of the base models to predict heights of large trees and trees outside the diameter range of the modeling data. Compared with the heights recorded for trees with maximum diameter, Equation 4 provided the lowest error rate for predicting the maximum height reported by Garman et al. (1995) (Table 5) and conformed to asymptotic behaviors of the most species considered in this study. The predictive ability of Equation 4 was also closest, in general, to the observed values for the five largest trees of each species. Equations 1 and 2 tended to grossly underestimate the asymptote. Based on these results, we chose Equation 4 as being best among the base equations.

The Chapman–Richards function (Equation 2) has been extensively used in describing height-age relationship. Huang et al. (1992) gave a cautionary note, however, stating that this function approaches the asymptote too quickly when there is a weak relationship between the dependent and independent variables. Hanus et al. (1999b) reported similar problems when applying the Chapman–Richards function to data from young Douglas-fir stands. Similar problems were observed in this study.

Evaluation of Expanded Equations

Biases for the expanded equations ranged from −0.1 m to +0.2 m (Table 6). None of the equations were unbiased for all species groups, whereas Equations 8–10, 14, and 16 were unbiased for 8 of the 10 species groups. The remaining equations proved to be unbiased for 4–7 of the 10 species groups.

Examining equations that provided the lowest RMSE for a species group (as indicated by the boldface values in Table 7) shows that Equations 13 and 14 had six species groups in which the equations resulted in the lowest RMSE values. The remaining equations provided lowest RMSE values for two to five species groups. Restricting the examination to unbiased equations, Equation 14 produced the lowest unbiased RMSE (as indicated by an asterisk on the values in Table 7) for six species groups, whereas Equation 13 proved best for five species groups. Based on these results, we chose Equation 14 as best among the expanded equations.

Table 5. Performance of the four equations examined using the average diameter [D5(cm)] and height [H5(m)] of the largest five trees in the modeling data and the maximum diameter [D_max(cm)] and height [H_max(m)] observed by Garman et al. 1995.

	Average of 5 largest trees		Observed by Garman *et al.*			Estimated height using		% difference using	
Species	D5(cm)	H5(m)	D_max(cm)	H_max(m)	Model	D5(cm)	D_Max(cm)	D5(cm)	D_max(cm)
Incense-cedar	130.4	49.3	174.8	54.7	1	46.0	55.2	6.6	−0.9
					2	46.0	55.6	6.6	−1.6
					3	37.3	41.8	24.2	23.6
					4	49.5	63.5	−0.5	−16.1
White/grand fir	103.5	53.5	136.4	61.9	1	45.0	48.1	15.9	22.3
					2	45.5	50.3	15.0	18.7
					3	45.2	50.1	15.5	19.1
					4	49.9	58.3	6.8	5.9
Sugar pine	115.0	45.9	188.5	59.7	1	41.6	46.4	9.4	22.2
					2	41.8	47.9	9.0	19.7
					3	41.6	49.1	9.2	17.8
					4	43.4	54.6	5.4	8.5
Ponderosa pine	105.8	47.1	147.6	52.4	1	40.8	42.6	13.3	18.8
					2	42.4	45.4	10.1	13.4
					3	46.1	52.5	2.2	−0.1
					4	47.0	54.6	0.4	−4.2
Douglas-fir	189.7	71.5	216.0	75.6	1	51.9	53.6	27.4	29.0
					2	52.1	54.4	27.2	28.0
					3	51.1	53.5	28.6	29.3
					4	66.1	71.2	7.6	5.8
Pacific madrone	63.9	27.8	126.0	37.5	1	23.7	27.8	14.6	25.9
					2	23.8	27.7	14.2	26.2
					3	23.2	28.4	16.6	24.3
					4	26.7	28.0	3.8	25.3
Golden chinkapin	46.2	23.7	75.7	21.9	1	21.8	28.0	7.8	−28.1
					2	22.1	30.4	6.8	−39.0
					3	20.8	26.4	12.1	−20.3
					4	23.9	36.4	−0.9	−66.3
Tanoak	42.6	21.5	79.5	21.9	1	20.9	28.8	2.7	−31.3
					2	20.9	28.7	2.6	−31.2
					3	19.0	20.3	11.6	7.2
					4	22.0	35.4	−2.0	−61.5
Canyon live oak	26.9	12.7	90.2	24.7	1	11.2	15.8	11.6	36.1
					2	11.3	16.4	11.1	33.6
					3	10.6	15.6	16.3	36.8
					4	12.2	28.5	4.3	−15.2
California black oak	83.1	27.8	115.2	32.9	1	18.1	44.0	34.9	−33.9
					2	18.3	44.4	34.3	−35.0
					3	19.7	37.1	29.0	−12.8
					4	19.5	46.2	29.9	−40.3

Table 6. Bias, in meters, by species and equation.

			Models with BAL and BA[a]				Models with CCFL and CCF				Models with CCFL and BA				Models with BAL and CCF			
Species	Plots (*n*)	Trees (*n*)	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Incense-cedar	110	623			0.1				0.1				0.1				0.1	
White/grand fir	115	907							0.1								0.1	
Sugar pine	56	124	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1	−0.1		−0.1	−0.1	−0.1	−0.1	−0.1	−0.1
Ponderosa pine	81	481			0.1						0.1	0.1	0.1					
Douglas-fir	339	7,953			0.1				0.1				0.1				0.1	
Pacific madrone	148	774																
Golden chinkapin	85	460								−0.1								−0.1
Tanoak	44	215	0.1	0.1	0.2	0.1	0.1	0.1	0.2	0.1	0.1	0.1	0.2	0.1	0.1	0.1	0.2	0.1
Canyon live oak	42	151									0.1							
California black oak	32	140	0.1	0.1	0.1				0.1		0.1		0.1		0.1	0.1	0.2	0.1

[a] BAL, basal area per hectare in larger trees; BA, basal area per hectare; CCFL, crown competition factor in larger trees; CCF, crown competition factor.

Comparison of Base to Expanded Equations

The inclusion of relative position (of a tree) and stand density measures to the base equation improved the predictive abilities of all base equations examined in this study. The expanded equations reduced the lowest RMSE for a species by a minimum of 6.1% for tanoak and canyon live oak to a maximum of 22% for the white fir and grand fir species group. An average reduction of 15% was found across all species groups. For all of the species groups, the effect of these measures on predicted H is as expected: they increase predicted height for a decrease in relative position in the stand (as indicated by

Table 7. Root mean square error (RMSE), in meters, by species and equation.[a]

			Models with BAL and BA[b]				Models with CCFL and CCF				Models with CCFL and BA				Models with BAL and CCF			
Species	Plots (n)	Trees (n)	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Incense-cedar	110	623	**2.3***	**2.3***	2.6	2.4	2.4	2.4	2.7	2.5	**2.3***	**2.3***	2.6	2.4	2.4	2.4	2.7	2.5
White/grand fir	115	907	3.8	3.8	3.8	3.8	**3.5***	**3.6***	3.7	**3.6***	**3.6***	**3.6***	3.7	**3.6**	3.7	3.7	3.8	3.7
Sugar pine	56	124	**3.4**	**3.4**	3.5	3.5	**3.4**	**3.4**	**3.4**	**3.4**	**3.4**	**3.4***	3.5	3.5	**3.4**	**3.4**	3.5	**3.4**
Ponderosa pine	81	481	3.5	3.5	3.4	3.5	3.4	3.4	**3.3***	3.4	3.4	3.4	3.4	3.5	3.4	3.4	3.4	3.4
Douglas-fir	339	7,953	3.8	3.8	3.9	3.8	**3.7***	**3.7***	3.8	**3.7***	**3.7***	**3.7***	3.8	**3.7***	3.8	3.8	3.9	3.8
Pacific madrone	148	774	2.6	2.6	2.6	2.6	**2.5***	**2.5***	2.6	2.6	**2.5***	**2.5***	2.6	2.6	2.6	2.6	2.7	2.7
Golden chinkapin	85	460	**2.1***	**2.1***	**2.1***	**2.1***	2.2	2.2	**2.1***	2.2	**2.1***	**2.1***	**2.1***	**2.1***	2.2	2.2	**2.1***	2.2
Tanoak	44	215	2.0	2.0	2.1	1.8	1.8	1.8	1.9	**1.7**	1.9	1.9	2.0	1.8	1.9	1.9	2.0	1.8
Canyon live oak	42	151	1.6	1.6	**1.5***	1.6	**1.5***	**1.5***	**1.5***	**1.5***	1.6	1.6	**1.5***	1.6	1.5*	**1.5***	**1.5***	**1.5***
California black oak	32	140	3.5	3.5	3.4	**3.3***	3.6	3.6	3.5	3.4	3.5	3.5	3.4	**3.3***	3.5	3.6	3.5	**3.3**

[a] Boldface indicates the absolute lowest RMSE value across all equations for a given species. An asterisk (*) indicates the lowest RMSE for unbiased equations only within a species.

[b] BAL, basal area per hectare in larger trees; BA, basal area per hectare; CCFL, crown competition factor in larger trees; CCF, crown competition factor.

Table 8. Parameter estimates (standard error of estimate) for base Equation 4 (i.e., using *D* only) by species.[a]

Species	Plots (n)	Trees (n)	a	b	c
Incense-cedar	110	623	14.7373 (2.647)	−15.4469(2.585)	−0.0728(0.016)
White/grand fir	115	907	5.3528 (0.191)	−7.3394(0.124)	−0.3505(0.099)
Sugar pine	56	124	4.893 (0.439)	−7.9038(0.944)	−0.4113(0.099)
Ponderosa pine	81	481	4.5503 (0.165)	−11.4582(1.28)	−0.5994(0.066)
Douglas-fir	339	7,953	5.7567 (0.077)	−6.7792(0.036)	−0.2795(0.008)
Pacific madrone	148	774	6.2681 (0.763)	−6.5453(0.673)	−0.189 (0.035)
Golden chinkapin	85	460	11.1522 (3.265)	−11.7201(3.286)	−0.1003(0.035)
Tanoak	44	215	10.2252 (3.004)	−10.4553(2.965)	−0.1018(0.036)
Canyon live oak	42	151	8.2443 (3.746)	−8.6686(3.664)	−0.1248(0.068)
California black oak	32	140	3.1434 (0.123)	−7.3864(2.065)	−0.850 (0.163)

[a] All estimated parameters were significantly different from 0 ($P < 0.005$).

Table 9. Parameter estimates (standard error of estimate) for expanded equation 14 (i.e., using *D*, relative position of a tree and stand density) by species. All estimated parameters were significantly different from 0 (*P* < 0.005).

Species	Plots (n)	Trees (n)	$a1$	$a2$	$a3$	b	c
Incense-cedar	110	623	53.518 (8.305)	0.076 (0.025)	0.2322 (0.047)	0.00829 (0.001)	1.0397 (0.023)
White/grand fir	115	907	37.9237 (2.370)	0.066 (0.011)	0.1431 (0.024)	0.0243 (0.002)	1.2451 (0.035)
Sugar pine	56	124	40.2518 (4.668)	0.115 (0.021)	0.0622 (0.060)	0.0213 (0.003)	1.3227 (0.096)
Ponderosa pine	81	481	41.1416 (4.232)	0.2126 (0.031)	0.2457 (0.039)	0.0165 (0.002)	1.2576 (0.048)
Douglas-fir	339	7,953	47.3816 (0.099)	0.0876 (0.004)	0.1021 (0.010)	0.0178 (0.001)	1.1088 (0.009)
Pacific madrone	148	774	24.9082 (3.892)	0.0633 (0.014)	0.0791 (0.030)	0.021 (0.005)	0.9674 (0.038)
Golden chinkapin	85	460	21.5789 (4.882)	0.0242 (0.015)	0.1253 (0.043)	0.0258 (0.007)	1.0374 (0.049)
Tanoak	44	215	30.1218 (6.423)	0.0345 (0.016)		0.0235 (0.007)	1.044 (0.046)
Canyon live oak	42	151	12.9442 (3.806)	0.0235 (0.011)		0.0427 (0.020)	1.0393 (0.093)
California black oak	32	140	12.2094 (2.706)	0.0315 (0.021)	0.1323 (0.048)	0.0443 (0.013)	1.1011 (0.112)

an increase in BAL or CCFL) or an increase in stand density (as indicated by an increase in BA or CCF). For two species, tanoak and canyon live oak, the parameter estimate on the density variable was of opposite sign from that expected. The density variable was therefore eliminated from the expanded equation for these two species.

The best expanded equation (Equation 14) uses CCFL as its measure of tree position, and it uses BA as a measure of stand density. An examination of Table 7 indicates that these two variables produced superior results more often than the other three combinations of variables examined in this study. These same variables have been found to be best in predicting the height to crown base for a tree in several studies from the Pacific Northwest (Ritchie and Hann 1987, Zumrawi and Hann 1989, Hanus et al. 2000, Hann et al. 2003). In all of these studies, a decrease in relative position in the stand or an increase in stand density results is smaller predicted crown lengths.

Although Equation 4 was found to be best when formulated with *D* alone, the expansion to include CCFL and BA resulted in a shift to Equation 2 as the best base equation. We suspect that the shift in asymptotic behavior of the model is caused by differences between equation forms in how fast predicted *H* approaches the asymptotic maximum *H* as *D* increases. To assess the asymptotic behavior of the four base equations, we divided predicted *H* at a *D* of 434 cm [the maximum observed by Waring and Franklin (1979)] by the maximum asymptotic *H* for the equations using the parameters for Douglas-fir. A ratio value near 1.0 would indicate strong asymptotic behavior. The resulting ratios were 0.68 for Equation 1, 0.68 for Equation 2, 0.72 for Equation 3, and 1.16 for Equation 4 (Table 6). Therefore, the shift from Equation 4 to Equation 2 corresponds to a shift from a weaker to a more strongly asymptotic base equation form, with the addition of the tree position and stand density variables. Adding stand density to the base equation does provide a means for localizing the regional *H-D* equation. Our past experiences leave us with the impression that the *H-D* equation for a stand is often more strongly asymptotic than a regional *H-D* equation.

Final Parameter Estimates

Once the equations were evaluated, all data were used to obtain parameter estimates (Tables 8 and 9). All estimated parameters were significantly different from 0 ($P < 0.005$). The final parameter estimates do vary considerably between species groups. Graphs of residuals from the refitted equations showed approximately homogenous variances over the full range of predicted values. Residual plots also indicated that tree height was well predicted across diameters.

Conclusions and Recommendations

The relative position and stand density measures used in this study are easily obtained and are available in most growth and yield models. When these variables were added to the base equation, the root mean square values were reduced by 15% on the average. Where possible, the use of the height–diameter function with these attributes is suggested. In summary, the suggested equation improves the accuracy of height prediction, ensures compatibility among the various estimates in a growth-and-yield model, and maintains projections within reasonable biological limits.

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