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Allometric equations for estimating aboveground biomass for common shrubs in northeastern California

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ABSTRACT

Selected allometric equations and fitting strategies were evaluated for their predictive abilities for estimating above ground biomass for seven species of shrubs common to northeastern California. Size classes for woody biomass were categorized as 1-h fuels (0.1–0.6 cm), 10-h fuels (0.6–2.5 cm), 100-h fuels (2.5–7.6 cm), and 1000-h fuels (greater than 7.7 cm in diameter). Three fitting strategies were evaluated - weighted nonlinear least squares regression (WNLS), seemingly unrelated regression (SUR), and multinomial log-linear regression (MLR) - to estimate individual shrub biomass as a function of crown area. The inclusion of the shrub height as a covariate did not increase the accuracy of prediction for all species. When MLR was used, on the average, RMSE values were reduced by 23.1% for the 1-h component, by 23.9% for the 10-h component, and by 45.6% for the leaf component for serviceberry when compared to SUR. Based on the residual plots and cross-validation fit statistics, MLR is recommended for estimating AGB for seven major shrub species in California. The equation coefficients are documented for future use.

Contents

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1. Introduction

Shrubs are important drivers of forest ecosystem productivity and diversity. Forest understory vegetation are ecologically important because shrubs, lichens, and mosses can have a direct effect on belowground processes such as decomposition, nutrient flow, and the accumulation of soil nutrients (Nilsson and Wardle, 2005). Many of the studies concerning forest biomass assessment that use of allometric equations have focused solely on the estimation of tree biomass (Beedlow et al., 2009; Temesgen et al., 2015). Although tree biomass is the principle sink of carbon sequestration in mature forests, it is also necessary to account for shrub biomass, as these woody plants play an active role in forest ecosystem productivity (Beedlow et al., 2009). Allometric shrub equations are also useful for shrubland ecosystems. Allometric equations for shrubs have been developed in the past (McGinnis et al., 2010; Vora, 1988; Elzein et al., 2011), however it is important to further contribute to this discipline as shrubs are important components of forest productivity and structure. A comprehensive assessment of total biomass will provide land managers and researchers with more reliable assessments of fuel loading, site productivity, and treatment effects (Návar et al., 2004). This research is necessary because allometric equations can contribute to the needs of forest modelers who are interested in assessing total and component biomass for carbon accounting and for fire modelers concerned with accumulating forest fuels and wildfire prevention.

Aboveground estimates of shrub biomass can also be effective indicators of the different stages of forest succession. Woody plants and shrubs growing under tree cover exhibit different form than from those growing in open areas (Vora, 1988; Paul et al., 2016). Shrubs growing under tree cover were found to be high in total volume and dead biomass but low in total and live biomass and number of twigs (Vora, 1988). Aboveground estimates of shrub biomass would be beneficial to wildlife managers and researchers interested in evaluating habitat for mammals and birds that may use these shrubs as cover or for nesting. The addition of shrub measurement data and allometric equations to existing forest tree biomass data can lead to a better understanding of forest productivity and diversity (Elzein et al., 2011; Chojnacky and Milton, 2008).

Aboveground shrub biomass equations are conducive to evaluating fuel loading behavior and fire prediction. Accurate assessments of wildfire behavior require quantitative estimates of available fuel weights by size category and condition. Categorizing fuel weights by size class is an important aspect in fire modeling (Murray and Jacobson, 1982). Size classes are determined by separating varying twig and branch diameters. Size classes for twigs and branches range from 0.1 to 0.6 cm, 0.6 to 2.5 cm, 2.5 to 7.6 cm, and greater than 7.6 cm. (Murray and Jacobson, 1982). These classes parallel with the 1-h, 10-h, 100-h, and 1000-h time lags defined in the National Fire Danger Rating (NFDR) system (Roussopoulos and Loomis, 1979). Time lag fuel categories are important to aboveground biomass studies because such categories are used in determining fire severity and intensity. Studies have shown that predictor variables, such as height and vegetation cover, can explain observed variation in total live fuel biomass (Sağlam et al., 2008). Having knowledge of whether biomass from understory shrubs is either alive or dead while knowing the time-lags of these fuel elements is important for fire behavior modelers who are concerned with fire rate of spread and intensity (Sağlam et al., 2008).

The lack of aboveground shrub biomass equations and the inability to predict shrub growth and accumulation has affected the accuracy of assessing forest fuel loads in California's forests. Little attention has been given to understory vegetation in these forests, primarily due to lack of economic value associated with these woody plants. However, it is important to obtain as much information about shrub growth and accumulation in these forests as these factors play a role in fire risk and likelihood over time. Shrub biomass models are needed to improve decision making in forest management, especially regarding fire risk and fuel treatments.

McGinnis et al. (2010) developed over 200 shrub and tree regression equations for Sierra Nevada, CA forests using stem diameter, basal area, crown diameter, and crown volume as covariates. The use of such covariates is common in the development of shrub biomass equations due to their ease of measurement in the field (McGinnis et al., 2010). The equations developed in this study produced estimates of biomass essential to predicting potential fire behavior, carbon sequestration rates, and in assessing wildlife habitat (McGinnis et al., 2010). Allometric equations that were developed for shrub species, such as greenleaf manzanita, deerbrush, and bush chinkapin, in the McGinnis et al. (2010) study also had total and component biomass equations developed for them in this study. This will be beneficial to researchers interested in proportional biomass as it relates to the composition of total biomass and with the development of fire behavior models for wildfire prevention.

Shrub biomass has been estimated for various aspects of forest management including fire risk management (Botequim et al., 2015; Roussopoulos and Loomis, 1979; Sağlam et al., 2008), carbon sequestration (Pasalodos-Tato et al., 2015, Zeng et al., 2010), ecological stresses or disturbances (Elzein et al., 2011), and wildlife habitat assessment (Grigal and Ohmann, 1977). Shrub biomass equations have been developed for forests and landscapes throughout the world, however, there is a need for site-specific, localized equations. The addition of shrub measurement data and allometric equations to existing forest tree biomass data can lead to a better understanding of forest productivity and diversity (Elzein et al., 2011). Allometric equations derived outside the forest ecosystem in question may not take into consideration spatial or temporal variability that may be present within that ecosystem (Ritchie et al., 2013).

The objectives of this article are to: (1) develop predictive equations for estimating aboveground biomass of seven species of shrubs using metrics easily measured in the field; and (2) assess the predictive abilities of three fitting strategies for estimating above ground biomass of woody shrubs common to the forests of northeastern California.

2. Materials and methods

2.1. Study area

The study area (Fig. 1) is located in Lassen National Forest, U.S. Forest Service (40°50′N, 121°00′W). The sampling elevation ranges from 1700 m to 2100 m. The annual precipitation varies from 584 mm to 1092 mm with a mean of 1041 mm. A majority of the precipitation comes in the form of snowfall between the months of November to April. The mean annual temperature is 7 °C, with a mean temperature of −6 °C in January and a mean temperature of 27 °C in August. Soils are classified as Typic Argixerolls and Typic Haploxerands, which were formed over colluvium, glacial till, or glacial outwash. Common plant associations within Lassen National Forest include the Jeffery pine (*Pinus jeffer-yi* [Grev. & Balf.])/white fir/greenleaf manzanita/snowbrush communities and the California red fir (*Abies magnifica* [A. Murr.])/white fir/bush chinkapin communities (found in higher elevations) (USDA, 2011).

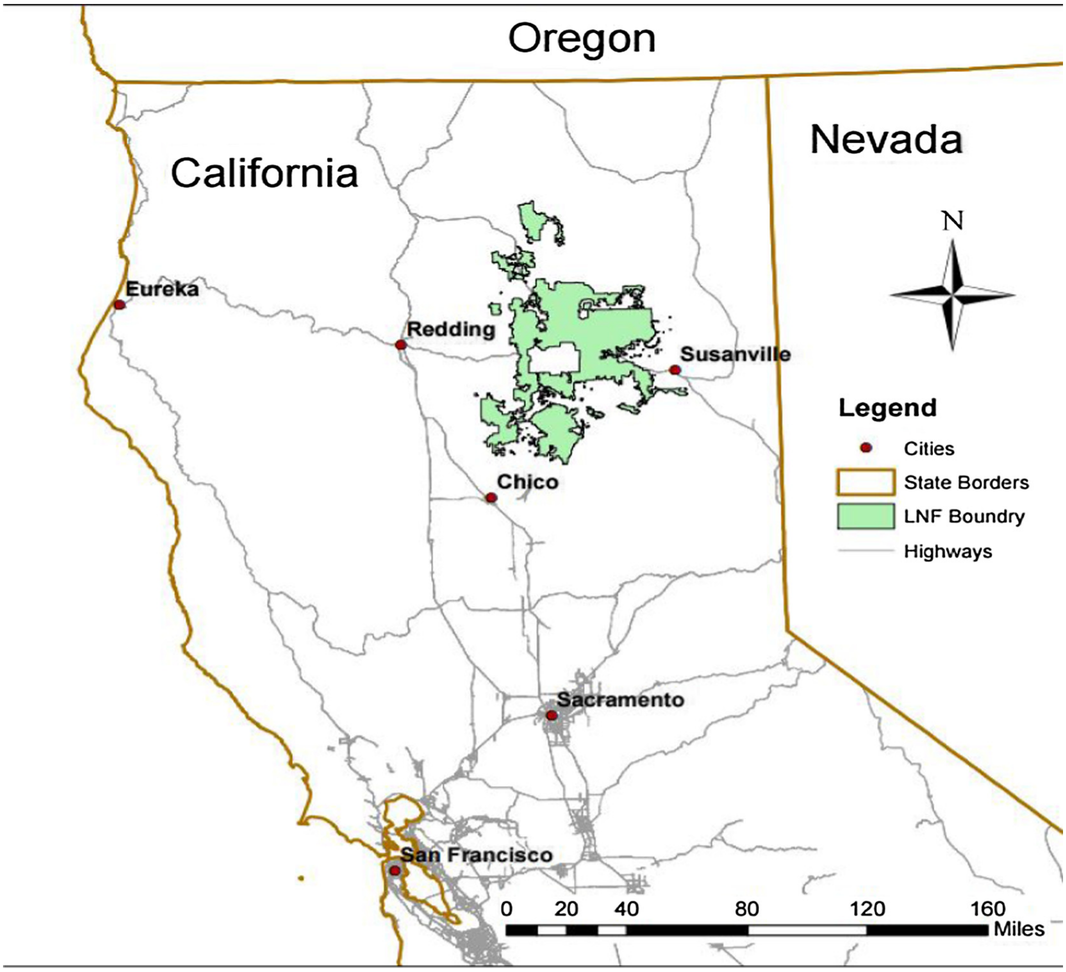

Fig. 1. Map depicting the study area located in Lassen National Forest, California.

Table 1
Common and scientific names, abbreviations, and total number of shrub samples.

Species (common)	Species (scientific)	n
Serviceberry	*Amalanchier alnifolia*	28
Greenleaf manzanita	*Arctostaphylos patula*	32
Bush chinkapin	*Castanopsis sempervirens*	20
Mountain whitethorn	*Ceanothus codulatus*	27
Deerbrush	*Ceanothus integerimus*	21
Snowbrush	*Ceanothus velutinus*	26
Ribes spp.	*Ribes* spp.	26

2.2. Data

The data were collected over the summers of 2011–2013. A total of 180 individual shrubs were sampled to fill a range of four height classes (0.1–0.5 m, 0.5–1.0 m, 1.0–1.5 m, and 1.5–2.0 m). A minimum of five shrubs per species within each height class was desired. Crews determined if the shrub was free to grow or not. Free to grow, for this study, was defined as whether or not the shrub crown was encroached by neighboring plants. Shrubs were only sampled if the crown dimensions could be readily observed due to the difficulty in measuring such dimensions without damaging the sample. If a tree or snag had fallen across a shrub, it was not selected for sampling.

Table 1 lists the shrub species sampled by common and scientific name and total number of samples obtained for each shrub. The seven shrubs species include: mountain whitethorn (*Ceanothus cordulatus* [Kellogg]), snowbrush (*Ceanothus velutinus* [Dougl. ex Hook.]), deerbrush (*Ceanothus integerrimus* [Hook. and Arn.]), bush chinkapin (*Castanopsis sempervirens* [Kellogg]), greenleaf manzanita (*Arctostaphylos patula* [Greene]), golden currant/-gooseberry (*Ribes* spp. [Pursh]), and serviceberry (*Amalanchier alnifolia* [Nutt.]). *Ribes* spp. includes combined observations of golden currant (*Ribes aureum*) and Sierra gooseberry (*Ribes roezlii*).

Shrubs were destructively sampled within the area of the Storrie Fire of 2000, but not exclusively. In some instances, shrub species within the desired size classes were not found, so samples were found on Blacks Mountain and Swain Mountain Experimental Forests (located within Lassen National Forest). Ecological knowledge and vegetation maps of the region were used to locate shrubs within this area. Field crews used a random number table to determine and set an arbitrary bearing and then walked that direction until a shrub that had the desired specifications (species; height within a specified height class) was located. Once a shrub with the desired specifications was located, its location was noted using handheld GPS devices, which allowing for location precision to within 3 to 6 m. A measure of crown width (cm) long (a measure of the horizontal crown width axis) and crown width short (a horizontal crown width perpendicular to the crown width long measurement) were obtained. Three measurements of height (cm) were taken for the tallest, second, and third tallest stems. Three measurements of the largest, second, and third largest basal diameters (cm) were also obtained at 10 cm aboveground and a count for the total number of stems was taken. Three measurements were obtained for both diameter because precision involving noncircular stems can be improved by averaging the three measurements. A total of eleven measurements were taken on each individual shrub.

Plant material was bagged by size class. Size classes used were adopted from the NFDR fuel classification system. Size classes include leaf (foliage), 1-h fuels (wood <0.64 cm in diameter), 10-h fuels (wood 0.64–2.54 cm in diameter), 100-h fuels (wood 2.54–7.62 cm in diameter), and 1000-h fuels (wood >7.62 cm in diameter) (Bradshaw et al., 1983). Total biomass is comprised of 1-h, 10-h, 100-h, 1000-h, and leaf biomass components (kg). Wood and leaves were bagged by size class and labeled denoting species, date, and size class of the material. Samples were stored in a dry room until the fall, when oven drying of the samples occurred.

Plant material was oven dried at 80 °C until weight was stabilized (generally 2–3 days). Weight of the leaves and 1-h biomass was processed first. Oven-dry biomass (g) for leaf biomass, 1-h, 10-h, 100-h, and 1000-h fuels were recorded. It should be noted that there was no plant material that was greater than 22.4 cm in diameter and very little of the recorded biomass fell into the 1000-h fuel class.

2.3. Data analysis

Crown area was considered as a predictor of shrub biomass due to accurate results obtained in past studies involving the estimation of shrub biomass (McGinnis et al., 2010; Zeng et al., 2010; Maraseni et al., 2005). Crown area (m^2) is calculated as: $CA = cwl * cws * \frac{\pi}{40{,}000}$ and is defined in this study as the area of a vertical projection of the crown to a horizontal plane (Uzoh and Ritchie, 1996).

It should be noted that increasing the sample size for each shrub species would have been beneficial to this analysis, as the size of the sample dictates the amount of information available for that shrub species and determines precision in the sample estimates. Increasing the amount of shrub species sampled in the field costs additional time and money, however, this may be desirable for obtaining a more robust sample of the population. Increasing the sample size may help to decrease sampling error, however, the allometry inherent within these shrubs will contribute to varying degrees of uncertainty within the estimates.

Several nonlinear growth models were considered to fit the data, including a multivariable (mean height (hh) and crown area) allometric model and a single variable power model that used mean height as a predictor of biomass. The model form for the multivariable allometric model was:

$$Y_{ij} = aX_{1ij}^{b}X_{2ij}^{c} + \varepsilon_{ij}, \quad i = 1, \ldots, m; \quad j = 1, \ldots, n_i,$$

where m is the number of components, n_i is the number of observations on the ith component, Y_{ij} is the jth shrub observation on the ith component, X_{1ij} is crown area for the jth shrub observation on the ith component, X_{2ij} is mean height for the jth shrub observation on the ith component, a, b, and c are regression coefficients, and ε_{ij} is the additive error term for jth shrub observation on the ith component, assumed to be normal (0, σ^2).

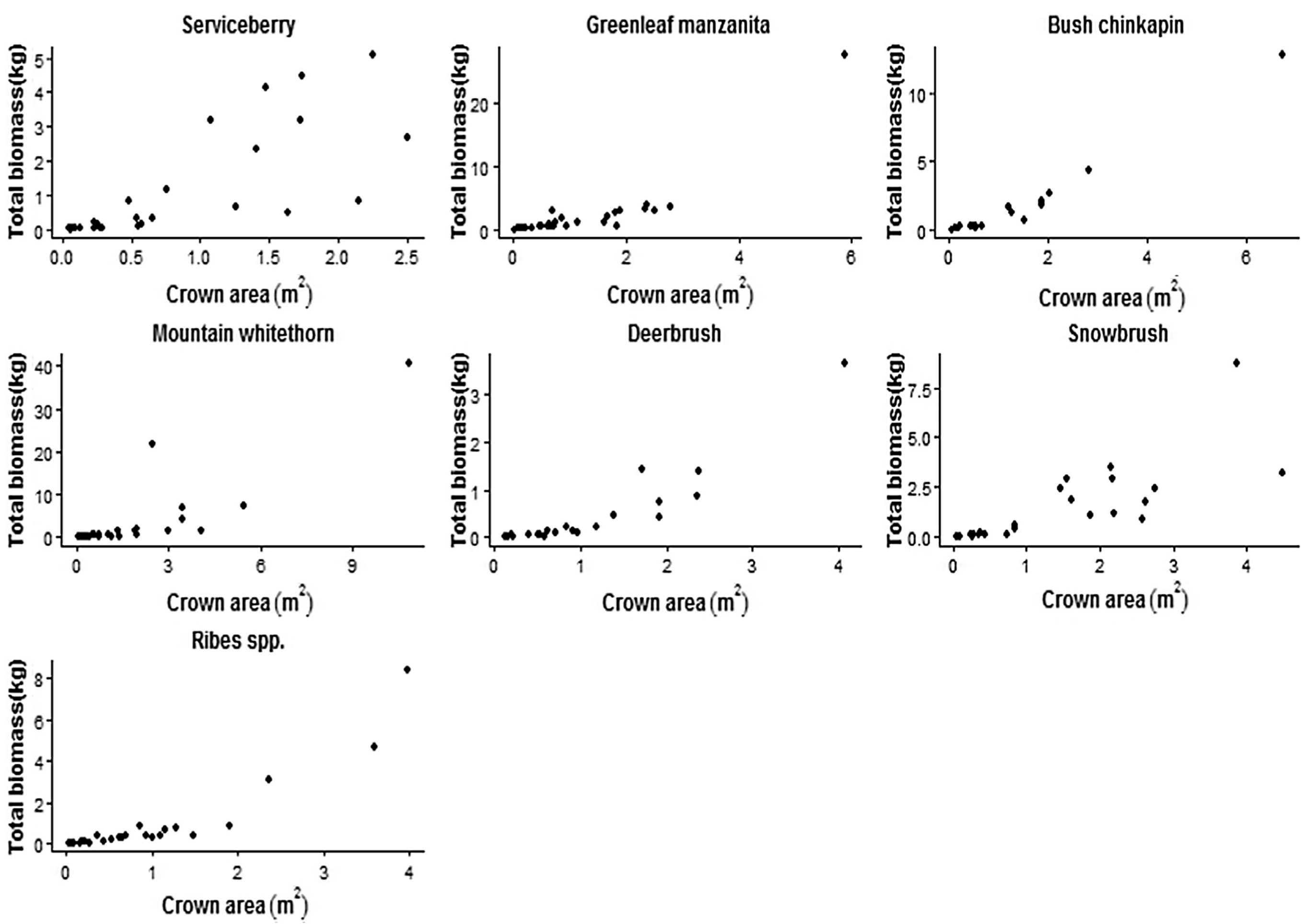

Fig. 2. Scatterplot depicting the relationship between crown area (*CA*) and total biomass for the seven species of shrubs.

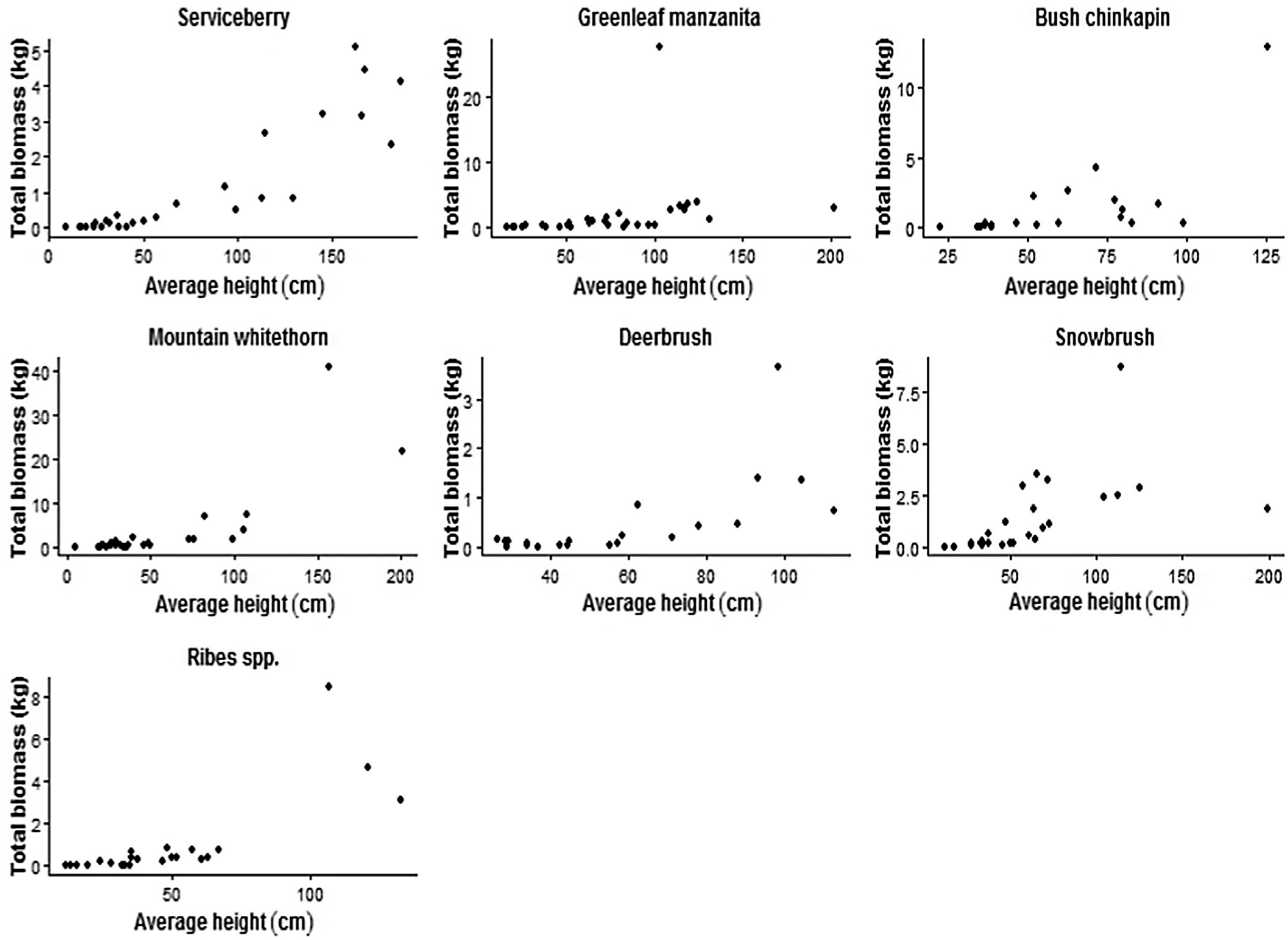

Fig. 3. Scatterplot depicting the correlation between mean height (*hh*) and total biomass for the seven species of shrubs.

A single variable allometric equation in the form of:

$$Y_{ij} = aX_{ij}^{b} + \varepsilon_{ij},$$

where $Y_{ij}, a, b,$ *and* ε_{ij} are as defined above and X_{ij} is crown area for the *j*th shrub observation on the *i*th component was also considered. A third model examined mean height alone as a predictor in lieu of crown area, with model form remaining the same as previously defined. The scatterplots shown in Figs. 2–4 depict a nonlinear relationship between crown area and total biomass and mean height and total biomass, respectively.Individual shrubs were subset by species. Starting values for nonlinear regression analysis were obtained for total, 1-h, and leaf biomass by fitting a linear log-log regression model in the form of:

$$log(Y_{ij}) = a + b\ log(X_{ij}) + \varepsilon_{ij},$$

where Y_{ij}, X_{ij}, *a*, *b*, and ε_{ij} are as previously defined. The starting values for the multivariable allometric model were obtained for total, 1-h, and leaf biomass by fitting a linear log-log model in the form of:

$$\log(Y_{ij}) = a + b\ \log(X_{1ij}) + c\ log(X_{2ij}) + \varepsilon_{ij},$$

where Y_{ij}, X_{1ij}, $a, b,$ and ε_{ij} are as previously defined, with *c* being a regression parameter, and X_{2ij} representing mean height. Starting values for all 10-h and 100-h biomass components for both the single and multivariable allometric models were obtained by fitting simple linear models in the form of:

$$Y_{ij} = a + bX_{ij} + \varepsilon_{ij} \text{ and } Y_{ij} = a + bX_{1ij} + cX_{2ij} + \varepsilon_{ij},$$

with all variables as defined previously. It should be noted that 1000-h biomass estimates were not calculated by NLS here due to the low occurrence (8 observations across all species) of this size class within each species. A simple linear log equation for the 1000-h component was instead created for shrubs that possessed these observations and is not reported.

Three fitting strategies, including weighted nonlinear least squares regression (WNLS), seemingly unrelated regression (SUR), and multinomial log-linear regression (MLR) were examined to estimate above ground biomass for seven shrub species common to northeastern California.

2.4. Weighted nonlinear least squares regression

Individual shrub biomass was fit using weighted nonlinear regression, $Y_{ij} = aX_{ij}^{b} + \varepsilon_{ij}$, where $\varepsilon_{ij} \sim \mathrm{N}(0, \frac{\hat{\sigma}2}{wt})$ and $wt = \frac{1}{CA_i^2}$. When heteroscedasticity is present within the regression model, the variance covariance matrix will have the following form:

$$\sigma^2(\varepsilon) = \begin{bmatrix} \sigma_1^2/wt & 0 & \cdots & 0 \\ \vdots & \sigma_2^2/wt & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_n^2/wt \end{bmatrix}$$

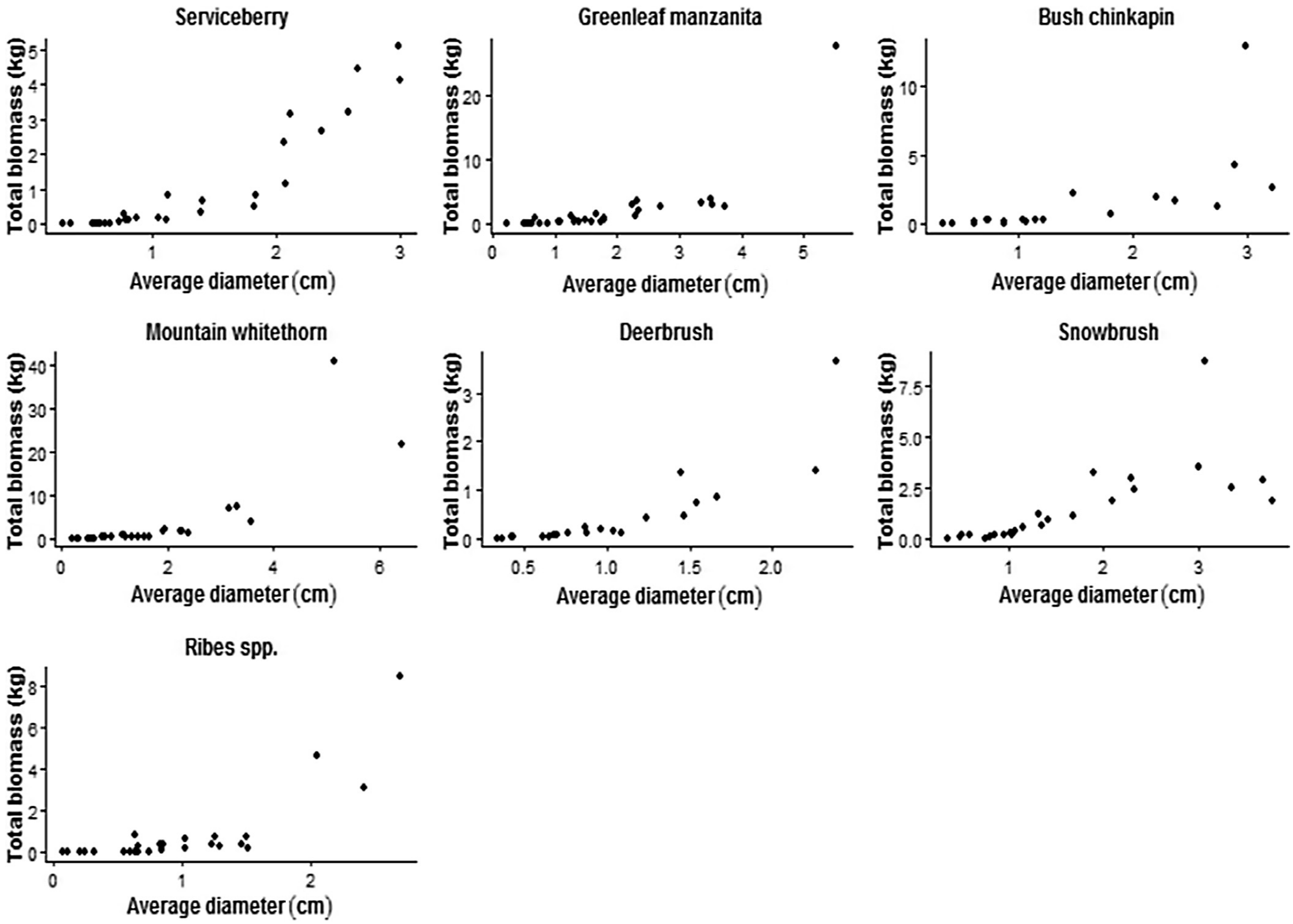

Fig. 4. Scatterplot depicting the correlation between mean diameter (*dd*) and total biomass for the seven species of shrubs.

where $\sigma_1^2, \sigma_2^2, \ldots \sigma_n^2$ represent differing variances for each error term. The weighted least squares estimate $\hat{\beta}_{WLS}$ is obtained as $(X'WX)^{-1}X'WY$, where W is the diagonal matrix of weights defined as

$$W = \begin{bmatrix} w_1 & 0 & \cdots & 0 \\ \vdots & w_2 & \ddots & \vdots \\ 0 & 0 & \cdots & w_n \end{bmatrix}$$

where $w_i = \frac{1}{\sigma_i^2}$ is equal to a value that is proportional to the variance of the error term (ε_{ij}).

Weighted regression in the form of $\frac{1}{CA}$ was employed to correct heteroscedasticity present in the data for all shrub species. The presence of local minima was checked by repeatedly fitting the model with varying initial values. Starting values obtained from the log-log models provided the smallest sum of squares for the parameter estimates.

Furnival's (Furnival, 1961) Index was calculated to compare different weighting options between the single variable and the multivariable power models. The index compares the fit of different weight functions and is defined as

$$FI = exp\left[\frac{P\sum_{i=1}^{n}(\ln ca)i}{n}\right] * RSE,$$

where P is a scalar that is half the value of the power used for the weight variable and RSE is residual standard error. Lower values of Furnival's Index indicate a better fit of the model.

Akaike's Information Criterion was used to measure the relative quality of the allometric models and is defined as:

$$AIC = -2lnL + 2q,$$

where (L) is the likelihood function for the model and q is equal to the number of parameter(s) the model contains. The lowest values of AIC were obtained from using the $\frac{1}{ca_i^2}$ weighting option across the seven shrub species.

2.5. Seemingly unrelated regression

Seemingly unrelated regression (SUR) was used to develop a system of equations for the nonlinear models. Statistical dependencies (simultaneous correlations) among sample data are accounted for by using nonlinear seemingly unrelated regression (Parresol, 2001; Kralicek et al., 2017). The structural equations for systems of nonlinear models were specified as

$$\begin{aligned} Y_l &= f_1(X_1, \beta_1) + \varepsilon_1 \\ Y_2 &= f_2(X_2, \beta_2) + \varepsilon_2 \\ &\vdots \\ Y_L &= f_L(X_L, \beta_L) + \varepsilon_L \\ Y_{total} &= f_{total}(X_1, X_2, \ldots, X_L, \beta_1, \beta_2, \ldots, \beta_L) + \varepsilon_{total}, \end{aligned}$$

where each component model contains its own independent variables and the total shrub regression is a function of all the independent variables used. $\varepsilon_1, \ldots, \varepsilon_L$ are independent across component, but may have cross-equation contemporaneous correlation.

The nonlinear model for total shrub biomass must be a combination of component biomass models to be additive (Parresol, 2001). Additivity of nonlinear equations is ensured by setting constraints on the regression coefficients. Since components are not independent of one another, there may be contemporaneous correlations. The strength of these correlations determines the increase in efficiency (Parresol, 2001).

SUR was performed to fit a nonlinear system of equations for biomass components within each species by defining individual components as

$$\begin{aligned}
&1\text{-}hour\ biomass = a_1 * ca^{b_1},\\
&10\text{-}hour\ biomass = a_2 * ca^{b_2},\\
&100\text{-}hour\ biomass = a_3 * ca^{b_3},\\
&1000\text{-}hour\ biomass = a_4 * ca^{b_4},\\
&Leaf\ biomass = a_5 * ca^{b_5},\\
&Total\ biomass = (a_1 * ca^{b_1}) + (a_2 * ca^{b_2}) + (a_3 * ca^{b_3}) + (a_4 * ca^{b_4}) + (a_5 * ca^{b_5}),
\end{aligned}$$

where a and b are regression coefficients obtained from previous model fits and CA is equal to crown area. The residuals obtained from a fitted simultaneous system of equations may exhibit correlation because the component biomasses come from the same shrub (Poudel and Temesgen, 2015). A SUR model was developed that combined all wood biomass together into a new component named *wood*. Leaf biomass was modeled separately from the wood component since total aboveground shrub biomass is composed of both wood and leaf biomass and because both components come from the same shrub. The model form is as follows:

$$\begin{aligned}
&1\text{-hour} + 10\text{-hour} + 100\text{-hour} + 1000\text{-hour} = (wood) = (a_1 * ca^{b_1}),\\
&Leaf\ biomass = (a_5 * ca^{b_5}),\\
&Total\ biomass = (a_1 * ca^{b_1}) + (a_5 * ca^{b_5}),
\end{aligned}$$

where $(a_1 * ca^{b_1})$ is equal to all wood biomass (*wood*) and $(a_5 * ca^{b_5})$ is equal to leaf biomass for each shrub species. Root mean squared error (RMSE) was obtained for both models after seemingly unrelated regression was applied.

2.6. Multinomial log-linear regression

Multinomial log-linear regression (MLR) was used to predict proportions of total shrub biomass found in 1-h, 10-h, 100-h, 1000-h, and leaf biomass components. The model form used for component proportions were

$$\begin{aligned}
p_{K1} &= \frac{1}{1 + e^{(a_1+a_2ca)} + e^{(b_1+b_2ca)} + e^{(c_1+c_2ca)} + e^{(d_1+d_2ca)}}\ \text{1-hour}\\
p_{K2} &= \frac{e^{(a_1+a_2ca)}}{1 + e^{(a_1+a_2ca)} + e^{(b_1+b_2ca)} + e^{(c_1+c_2ca)} + e^{(d_1+d_2ca)}}\ \text{10-hour}\\
p_{K3} &= \frac{e^{(b_1+b_2ca)}}{1 + e^{(a_1+a_2ca)} + e^{(b_1+b_2ca)} + e^{(c_1+c_2ca)} + e^{(d_1+d_2ca)}}\ \text{100-hour}\\
p_{K4} &= \frac{e^{(c_1+c_2ca)}}{1 + e^{(a_1+a_2ca)} + e^{(b_1+b_2ca)} + e^{(c_1+c_2ca)} + e^{(d_1+d_2ca)}}\ \text{1000-hour}\\
p_{KL} &= \frac{e^{(d_1+d_2ca)}}{1 + e^{(a_1+a_2ca)} + e^{(b_1+b_2ca)} + e^{(c_1+c_2ca)} + e^{(d_1+d_2ca)}}\ Leaf,
\end{aligned}$$

where p_{K1}, p_{K2}, p_{K3}, p_{K4}, and p_{KL} are proportions of total shrub biomass found in 1-h, 10-h, 100-h, 1000-h, and leaf biomass, respectively; CA = crown area; and a_i, b_i, c_i, and d_i ($i = 1, 2$) are model parameters (Poudel and Temesgen, 2015).

Predicted proportions were applied to observed total aboveground shrub biomass in order to obtain predicted biomass estimates in different components by applying the function multinom in the package nnet in R version 3.2.2. (Poudel and Temesgen, 2015). MLR was used to compare individual proportions of biomass components separately and for when different combinations of fuel classes were combined, namely the combination of 100-h and 1000-h components and the grouping of 10-h and 100-h components together while omitting the 1000-h component from the fitting process.

2.7. Parameter estimation

Equation parameters in MLR were estimated with the Gauss-Newton optimization technique in a weighted, nonlinear least squares procedure in R. After comparing different weighting schemes, a weight of $1/CA^2$ was used based on the Furnival Index. Initial approximations for each parameter were obtained from linear transformation of the equations, where possible. The starting value of each parameter was varied to find a global minimum, and the run with the smallest MSE was chosen as providing the final parameter estimates. The assumption of homoscedasticity of the weighted residuals was tested using the Goldfield-Quandt test (Goldfield and Quandt, 1965). The test indicated homogenous variances over the full range of predicted values at a 0.05 a level.

2.8. Fitting strategy comparison and selection

The performance of the three fitting strategies were examined using residual plots and a jackknife, exclude-one-plant validation technique (Stone, 1974). One shrub from each species was excluded from the data set, and the selected models were fitted using the rest of the data for that shrubs species. Then, the models were used to predict the AGB of the excluded shrub. The same process was repeated for every shrub of that species in the data set.

The following jackknife cross-validation statistic was applied to the cumulative data from the excluded shrubs to evaluate the performance of the three fitting strategies:

(1) Prediction bias

$$Bias = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \widehat{Y}_i),$$

where Y_i is the observed ith shrub, $\widehat{Y}_i$ is the predicted unweighted value of the ith shrub, and n is equal to sample size. In statistics, bias is referred to as the difference between the true value of an unknown parameter of interest and the expected value of its estimator. In the context of this study, bias is defined as the mean difference between the measured value and the predicted value of the variable of interest (Poudel and Temesgen, 2015).

(2) Root mean squared prediction error (RMSPE)

$$RMSPE = \frac{1}{n}\sum_{i=1}^{n}\sqrt{(Y_i - \widehat{Y}_i)^2},$$

where Y_i is the observed ith shrub, $\widehat{Y}_i$ is the predicted unweighted value of the ith shrub, and n is equal to sample size.

(3) Relative root mean squared prediction error (RRMSPE)

$$RRMSPE = \frac{RMSPE}{\bar{y}} * 100,$$

where $\bar{y}$ is the mean value of the response variable being modeled (biomass component) and RMSPE is as described above (unweighted). Relative root mean square prediction error is a relative error expressed as a percentage and is obtained after cross validation has been performed. RRMSPE evaluates the relative closeness of the predictions to the actual values.

Table 2
Summary statistics used for data analysis. Kt = total biomass, K1 = 1-h biomass, K2 = 10-h biomass, K3 = 100-h biomass, and KL = leaf biomass (kg).

Response						Predictor				
Species	Variable	Minimum	Maximum	Mean	Std Dev	Variable	Minimum	Maximum	Mean	Std Dev
Serviceberry (kg)	Kt	0.008	5.08	1.09	1.57	Height (cm)	10.00	200.00	77.79	61.83
	K1	0.004	1.44	0.34	0.45					
	K2	0.000	2.47	0.56	0.84	Crown area (m^2)	0.05	2.51	0.81	0.76
	K3	0.000	0.59	0.71	0.15					
	KL	0.002	0.39	0.09	0.11	Diameter (cm)	0.39	3.47	1.62	0.99
Manzanita (kg)	Kt	0.080	3.92	1.09	1.24	Height (cm)	19.00	190.00	77.13	39.32
	K1	0.004	0.62	0.20	0.20					
	K2	0.000	1.99	0.42	0.51	Crown area (m^2)	0.02	2.78	0.91	0.84
	K3	0.000	1.79	0.21	0.43					
	KL	0.004	0.97	0.26	0.28	Diameter (cm)	0.54	6.71	2.11	1.45
Chinkapin (kg)	Kt	0.023	4.27	0.87	1.15	Height (cm)	19.00	105.00	63.00	24.31
	K1	0.007	1.21	0.22	0.31					
	K2	0.000	1.96	0.32	0.48	Crown area (m^2)	0.05	2.83	0.87	0.81
	K3	0.000	0.56	0.09	0.18					
	KL	0.012	0.82	0.24	0.27	Diameter (cm)	0.45	3.91	1.82	1.02
Whitethorn (kg)	Kt	0.003	21.66	1.75	4.41	Height (cm)	6.00	193.00	52.08	41.38
	K1	0.001	2.76	0.49	0.76					
	K2	0.000	6.34	0.56	1.35	Crown area (m^2)	0.04	4.09	1.28	1.20
	K3	0.000	8.86	0.41	1.76					
	KL	0.002	1.88	0.18	0.38	Diameter (cm)	0.26	8.88	1.86	1.74
Deerbrush (kg)	Kt	0.012	3.65	0.48	0.84	Height (cm)	31.00	122.00	64.71	28.03
	K1	0.008	2.01	0.23	0.44					
	K2	0.000	0.93	0.15	0.24	Crown area (m^2)	0.13	4.07	1.13	0.98
	K3	0.000	0.10	0.01	0.03					
	KL	0.002	0.70	0.09	0.16	Diameter (cm)	0.49	2.60	1.20	0.61
Snowbrush (kg)	Kt	0.013	8.72	1.37	1.90	Height (cm)	13.00	194.00	64.46	38.76
	K1	0.006	2.00	0.15	0.47					
	K2	0.000	3.72	0.78	0.80	Crown area (m^2)	0.05	4.49	1.34	1.22
	K3	0.000	0.78	0.15	0.24					
	KL	0.006	1.84	0.33	0.41	Diameter (cm)	0.52	4.19	1.87	0.94
Ribes spp. (kg)	Kt	0.003	8.46	0.85	1.87	Height (cm)	12.00	159.00	51.81	34.88
	K1	0.002	2.08	0.25	0.46					
	K2	0.000	3.16	0.34	0.79	Crown area (m^2)	0.03	3.99	0.92	1.04
	K3	0.000	0.41	0.04	0.09					
	KL	0.001	2.23	0.17	0.05	Diameter (cm)	0.09	3.46	1.17	0.84

In order to observe model behavior, plots of predicted values versus observed biomass values were created. Plots were also created showing the relationship between the fitted model values against the model's standardized residuals. All statistical analysis was performed in R (R Core Team, 2016), except for SUR, which was performed using SAS software.

3. Results

Allometric equations were formulated across seven different species of shrubs using weighted nonlinear least squares regression. Five individual biomass components were used to develop these equations so that allometric relationships within each species may be better understood. The results provide evidence that the power model provides unbiased estimates for most species and biomass components, but provides somewhat inconsistent measures of accuracy due to variability within shrub species and biomass components. SUR provided efficient estimates for most species and biomass components. MLR resulted in unbiased estimates across all species and components and provided low values of RMSE.

Data were organized into two separate categories: shrub green weights (weights of biomass size classes obtained in the field (g)) and shrub dry weights (weights of biomass size classes after oven drying). Serviceberry had the greatest value for mean height (77.8 cm) with a minimum height of 10 cm and a maximum height of 200 cm. Greenleaf manzanita possessed the largest mean basal diameter (2.1 cm) with a minimum basal diameter of 0.5 cm and a maximum basal diameter of 6.7 cm. Snowbrush had the greatest mean value of crown area (1.3 m^2) with a minimum crown area equal to 0.1 m^2 and a maximum crown area equal to 4.5 m^2. Mountain whitethorn possessed the largest mean total biomass (1.8 kg) with minimum and maximum weights equal to 0.1 kg and 21.7 kg, respectively. Deerbrush had the lowest mean total biomass (0.5 kg) with minimum and maximum weights equal to 0.1 kg and 8.5 kg, respectively (Table 2).

Fig. 2 depicts the relationship between total shrub biomass and crown area. Although serviceberry had a smaller observed crown area, there is some variability between total biomass and crown area. Total shrub biomass increases as crown area increases for all shrubs. Figs. 3 and 4 illustrate the relationship between total shrub biomass and mean height and mean diameter for all shrub species. Total shrub biomass increases as mean height increases, although the increase in total biomass is not well defined for some shrub species. For example, for greenleaf manzanita, the increase in total biomass in relation to mean height increase tends to be less pronounced than other shrubs. Bush chinkapin exhibits variability in total shrub biomass when mean height is 70 cm, with biomass increasing when mean height is less than 70 cm and decreasing when mean height is greater than 70 cm. The scatterplots help to depict the variability that is present in total shrub biomass when mean height is considered as a covariate.

3.1. Weighted nonlinear least squares regression

Table 3 provides parameter estimates for total biomass for all shrub species using an unweighted, $\frac{1}{CA_i}$ and $\frac{1}{CA_i^2}$, weighting options. RMSPE tends to increase slightly for five of the species when the

Table 3
Differences in weighting options for total biomass by species using crown area (CA) as a covariate. Standard errors of the coefficients are shown in parentheses. a and b are regression coefficients, Var = estimated variances for a and b, and RMSPE = root mean square prediction error (kg).

Species	Coefficients (standard error)		Estimated variances		Goodness of fit statistics		
Serviceberry	a	b	Var($\hat{a}$)	Var($\hat{b}$)	RMSPE	Bias	Furnival's Index
Unweighted	1.4002 (0.2919)	1.0682 (0.3031)	0.0852	0.0919	1.1365	−0.0818	0.18
1/CA	1.2289 (0.2108)	1.3223 (0.2496)	0.0444	0.0623	1.1498	−0.023	0.14
1/CA^2	1.1322 (0.1615)	1.5412 (0.1897)	0.0261	0.036	1.1862	−0.0301	0.13
Greenleaf manzanita	a	b	Var($\hat{a}$)	Var($\hat{b}$)	RMSPE	Bias	Furnival's Index
Unweighted	1.1652 (0.1726)	1.0945 (0.1903)	0.0297	0.0362	0.6714	0.0003	0.13
1/CA	1.1627 (0.1396)	1.1036 (0.1563)	0.0195	0.0244	0.6628	−0.0013	0.13
1/CA^2	1.1400 (0.1457)	1.2231 (0.1247)	0.0212	0.0156	0.6675	−0.0361	0.15
Bush chinkapin	a	b	Var($\hat{a}$)	Var($\hat{b}$)	RMSPE	Bias	Furnival's Index
Unweighted	0.7296 (0.0951)	1.6834 (0.1552)	0.0091	0.0241	0.3196	0.0425	0.06
1/CA	0.7902 (0.0869)	1.5718 (0.1440)	0.0076	0.0206	0.3231	0.0233	0.06
1/CA^2	0.8618 (0.0857)	1.3569 (0.1145)	0.0073	0.0131	0.3857	0.0341	0.08
Mountain whitethorn	a	b	Var($\hat{a}$)	Var($\hat{b}$)	RMSPE	Bias	Furnival's Index
Unweighted	1.5047 (1.1227)	0.9931 (0.6759)	1.2605	0.4568	4.364	−0.259	1.98
1/CA	1.0549 (0.6983)	1.3573 (0.6166)	0.4876	0.3802	4.4242	−0.1135	1.26
1/CA^2	0.8145 (0.4136)	1.6497 (0.4956)	0.1711	0.2457	4.2574	−0.0281	0.80
Deerbrush	a	b	Var($\hat{a}$)	Var($\hat{b}$)	RMSPE	Bias	Furnival's Index
Unweighted	0.2181 (0.0433)	1.9995 (0.1579)	0.0019	0.0249	0.2919	0.0419	0.12
1/CA	0.2265 (0.0424)	1.9662 (0.1634)	0.0018	0.0267	0.2653	0.0259	0.09
1/CA^2	0.2436 (0.0404)	1.8852 (0.1720)	0.0016	0.03	0.2779	0.0227	0.08
Snowbrush	a	b	Var($\hat{a}$)	Var($\hat{b}$)	RMSPE	Bias	Furnival's Index
Unweighted	0.8938 (0.3038)	1.2074 (0.2914)	0.0922	0.083	2.0318	−0.1937	0.63
1/CA	0.8153 (0.1882)	1.2987 (0.2175)	0.0354	0.0473	1.6128	−0.0676	0.37
1/CA^2	0.7583 (0.1029)	1.3905 (0.1371)	0.0106	0.0188	1.4027	−0.0289	0.25
Ribes	a	b	Var($\hat{a}$)	Var($\hat{b}$)	RMSPE	Bias	Furnival's Index
Unweighted	0.2963 (0.0829)	2.3462 (0.2140)	0.0069	0.0458	0.8313	0.1586	0.06
1/CA	0.3790 (0.0768)	2.1499 (0.1629)	0.0059	0.0265	0.7263	0.0643	0.05
1/CA^2	0.5070 (0.0740)	1.8707 (0.1389)	0.0055	0.0193	0.8228	0.0484	0.04

weight $\frac{1}{ca_i^2}$ was employed. The increase in RMSPE was nominal, with the largest gap between $\frac{1}{CA_i}$ and $\frac{1}{CA_i^2}$ being 0.10 kg (*Ribes* spp.). Bias was negligible for all shrub species and there was a noticeable decrease in the standard error of the coefficients as heavier weighting options were employed. Furnival's index was lowest for serviceberry, mountain whitethorn, deerbrush, snowbrush, and *Ribes* spp., indicating a better model fit. Akaike's Information Criterion (AIC) was applied to determine best model fit for the three weighting options. Although the *b* parameter estimates for serviceberry, greenleaf manzanita, mountain whitethorn, and snowbrush increase when this weight is applied, the standard errors of the coefficients are lowest when compared to the other weighting schemes. The increases in the *b* parameter estimate values are likely caused by high amounts of variability found within the data, especially for larger shrubs like mountain whitethorn and snowbrush. Model fits were deemed best when weights were equal to $\frac{1}{ca_i^2}$. Using the single variable power model with crown area as a predictor resulted in convergence for the 10-h and 100-h biomass components for all species.

The power model performed well in terms of bias with values remaining very low for almost all of the biomass components. RMSPE and RRMSPE varied between species and components, which was somewhat expected given the sporadic sprouting and branching characteristics that each of these shrubs possesses. Overall, the power model exhibited the most stability within the 1-h and leaf biomass components for all species, with the exception of mountain whitethorn, using weighted nonlinear least squares regression (Fig. 5). Inflated values of RRMSPE results indicate that the model has a large amount of variation between the predicted and observed biomass values. The precision of the power model for mountain whitethorn can therefore be defined as poor across all biomass components. All parameter estimates and goodness of fit statistics for each shrub species and biomass component are found in Table 4.

All RRMSPE values increased sharply from the 10-h component to the 100-h component. There is a 63.25 kg difference between the 10-h and 100-h biomass components. A total of 160 shrubs had biomass that fell within the 10-h component diameter range whereas only 72 shrubs had biomass that fell within the 100-h component diameter range. The resulting RRMSPE values from each species' model fit explain that the model is unstable for the 100-h biomass component.

3.2. Seemingly unrelated regression

Seemingly unrelated regression (SUR) was used to construct aboveground biomass equations for all shrub species. A combined wood SUR model was constructed that grouped all woody biomass into one component called *wood*. Tables 5 and 6 depict parameter estimates and standard errors for both the combined wood and individual component SUR models. Overall, combining wood into one category caused total biomass RMSE to decrease when compared to the components fit in the individual component model. This was most apparent with serviceberry and mountain whitethorn observations, where total biomass RMSE decreased by 0.19 kg and 2.08 kg for each species, respectively. These species had large values of total biomass and had observations that fell within the 100-h and 1000-h biomass components. Combining the wood observations helped decrease error somewhat, however, inference concerning individual fuel classes is also lost in this process.

3.3. Multinomial log-linear regression

Results from MLR unbiasedly predicted component proportions for all shrub species. Table 7 provides RMSE and RRMSE for all biomass components, with the 1-h biomass component being used as the reference equation. RMSE and RRMSE were highest within the 100-h biomass component. For species that had observations that

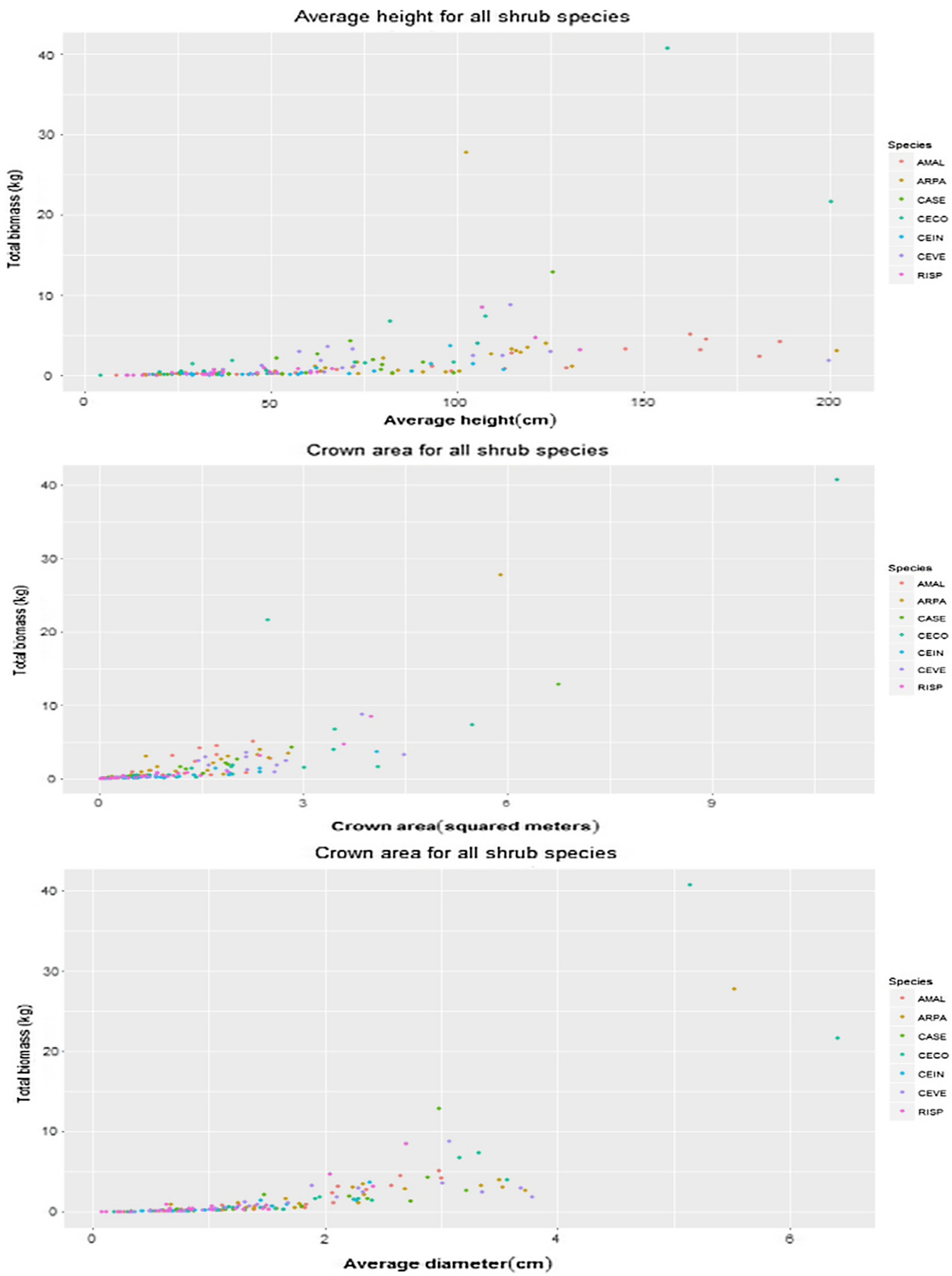

Fig. 5. Scatter plots of total biomass (Kt) vs average height, crown area, and average diameter for all shrub species.

fell within the 1000-h component, RMSE and RRMSE were equally or more inflated. Mean RRMSE across all species and the 100-h biomass component is 140.43%.

RMSE and RRMSE values improved when the 100-h and 1000-h components were combined, as seen in Table 8. When these two components are combined, the RRMSE for species that had 1000-h observations decreased within the 100-h + 1000-h component. This was most evident with serviceberry and snowbrush. Mean RRMSE across all species within the 100-h + 1000-h component is 119.64%.

The 10-h and 100-h biomass components were grouped and the 1000-h component was dropped from the fitting process, as depicted in Table 9. Shrub species that did not contain 1000-h observations were unbiased, however, results were biased for serviceberry, mountain whitethorn, snowbrush, and *Ribes* spp. While RRMSE values for this grouping method are much more reasonable when compared to the results shown in Table 7, the bias% apparent in species that had 1000-h observations that were dropped from the analysis are concerning. Mean RRMSE for the 100-h + 1000-h component while omitting the 1000-h observations is 35.22%.

Table 4
Weighted nonlinear least squares parameter estimates for total, 1-h, 10-h, 100-h, and leaf biomass components (kg) by species for power model with crown area as a predictor. Standard errors of the coefficients are shown in parentheses. Weights = $1/CA^2$, a and b are regression coefficients, Var = estimated variances for a and b, RMSPE = root mean square prediction error (kg), RRMSPE = relative root mean square prediction error, and FI = Furnival's Index.

Species	Coefficients (standard error)		Goodness of fit statistics			
Serviceberry	a	b	RMSPE	RRMSPE	Bias	FI
Total biomass	1.1322 (0.1615)	1.5412 (0.1897)	1.19	108.65	−0.03	0.13
1-h	0.3867 (0.0454)	1.3643 (0.1344)	0.34	98.17	−0.01	0.04
10-h	0.5363 (0.0920)	1.7063 (0.2447)	0.63	27.77	−0.01	0.07
100-h	0.0799 (0.0265)	1.4352 (0.4079)	0.16	219.31	−0.01	0.02
Leaf	0.1017 (0.0114)	1.2857 (0.1176)	0.08	96.65	0.00	0.01
Greenleaf manzanita	a	b	RMSPE	RRMSPE	Bias	FI
Total biomass	1.1627 (0.1396)	1.1036 (0.1563)	0.67	61.04	−0.04	0.15
1-h	0.2243 (0.0168)	0.9652 (0.0924)	0.09	43.93	0.00	0.02
10-h	0.4102 (0.0437)	1.2899 (0.1407)	0.23	96.07	0.03	0.04
100-h	0.2255 (0.0892)	1.1253 (0.5173)	0.40	187.39	0.00	0.08
Leaf	0.2910 (0.0276)	0.9703 (0.1172)	0.16	60.99	0.00	0.03
Bush chinkapin	a	b	RMSPE	RRMSPE	Bias	FI
Total biomass	0.8618 (0.0857)	1.3569 (0.1145)	0.39	44.50	0.03	0.08
1-h	0.2222 (0.0264)	1.2219 (0.1203)	0.17	77.42	0.02	0.02
10-h	0.2971 (0.0356)	1.5678 (0.1552)	0.21	138.55	0.01	0.03
100-h	0.0525 (0.0230)	2.3048 (0.5512)	0.45	511.61	−0.08	0.01
Leaf	0.2570 (0.0398)	1.0265 (0.1247)	0.13	54.44	0.01	0.03
Mountain whitethorn	a	b	RMSPE	RRMSPE	Bias	FI
Total biomass	0.8145 (0.4136)	1.6497 (0.4956)	4.26	243.62	−0.03	0.80
1-h	0.2855 (0.0513)	1.3951 (0.1820)	0.65	133.64	−0.02	0.11
10-h	0.2217 (0.1283)	1.8231 (0.5433)	1.38	338.02	−0.04	0.23
100-h	0.1719 (0.1877)	1.7587 (1.0397)	1.90	465.91	−0.04	0.35
Leaf	0.1080 (0.0311)	1.3660 (0.2909)	0.37	206.61	−0.01	0.07
Deerbrush	a	b	RMSPE	RRMSPE	Bias	FI
Total biomass	0.2436 (0.0404)	1.8852 (0.1720)	0.28	57.52	0.02	0.08
1-h	0.1109 (0.0204)	1.9640 (0.1853)	0.23	99.41	0.03	0.04
10-h	0.0759 (0.0149)	1.8593 (0.2056)	0.12	46.82	−0.01	0.03
100-h	0.0084 (0.0035)	1.7097 (0.45369)	0.07	479.44	−0.01	0.01
Leaf	0.0452 (0.0071)	1.8413 (0.1669)	0.08	87.47	0.00	0.01
Snowbrush	a	b	RMSPE	RRMSPE	Bias	FI
Total biomass	0.7583 (0.1029)	1.3905 (0.1371)	1.40	102.23	−0.03	0.25
1-h	0.2164 (0.0293)	1.2819 (0.1342)	0.35	97.26	−0.01	0.07
10-h	0.2506 (0.0495)	1.5353 (0.1970)	0.64	44.72	−0.02	0.11
100-h	0.0675 (0.0209)	1.5998 (0.3058)	0.23	157.92	−0.01	0.04
Leaf	0.2106 (0.0182)	1.1937 (0.0820)	0.26	77.83	0.00	0.05
Ribes	a	b	RMSPE	RRMSPE	Bias	FI
Total biomass	0.5070 (0.0740)	1.8707 (0.1389)	0.82	96.45	0.05	0.04
1-h	0.2176 (0.0200)	1.3731 (0.0940)	0.19	76.43	0.02	0.01
10-h	0.1550 (0.0323)	2.1634 (0.1818)	0.20	26.77	0.00	0.02
100-h	0.0307 (0.0092)	1.4649 (0.3138)	0.09	255.66	0.00	0.01
Leaf	0.0885 (0.0214)	2.0391 (0.2185)	0.36	204.75	0.02	0.01

Table 5
Parameter estimates and their standard errors for the combined wood seemingly unrelated regression (SUR) models for all shrub species. The combined wood model consists of all wood components combined (wood = 1-h + 10-h + 100-h + 1000-h) and leaf biomass is modeled separately. a1, a2, b1, and b2 are parameter estimates.

Serviceberry			Greenleaf manzanita			Bush chinkapin			Mountain whitethorn		
Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error
a1	1.1908	0.2656	a1	0.8763	0.151	a1	0.4373	0.0732	a1	1.3273	1.0029
b1	1.2164	0.3092	b1	1.1324	0.2201	b1	1.9894	0.1916	b1	1.0183	0.6796
a2	0.1131	0.0182	a2	0.2921	0.0382	a2	0.2676	0.0385	a2	0.1675	0.1144
b2	0.9344	0.2263	b2	0.9529	0.1707	b2	1.0975	0.1871	b2	0.8386	0.6374
Deerbrush			Snowbrush			*Ribes*					
Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error			
a1	0.1749	0.0361	a1	0.6838	0.2521	a1	0.377	0.0705			
b1	2.011	0.1638	b1	1.2081	0.3162	b1	1.9709	0.1466			
a2	0.0298	0.0053	a2	0.2101	0.0574	a2	0.0001	1.5E−05			
b2	2.2465	0.1394	b2	1.2066	0.234	b2	8.8828	1.1044			

However, the omission of the 1000-h component from the analysis caused species that actually had observed values of the 1000-h component to become biased (mean bias% equal to −4.43% across species with 1000-h observations).

The MLR method produced RMSE values that were lower than both the combined wood and individual component SUR methods. Using serviceberry and mountain whitethorn as examples, values of RMSE were considerably lower for these three species across

Table 6
Parameter estimates and their standard errors for the individual component seemingly unrelated regression (SUR) models for all shrub species. The individual component model is comprised of all biomass components modeled separately. a1, a2, b1, and b2 are parameter estimates.

Serviceberry			Greenleaf manzanita			Bush chinkapin			Mountain whitethorn		
Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error
a1	0.4649	0.0336	a1	0.2078	0.0202	a1	0.1317	0.0277	a1	0.4227	0.0542
b1	1.2538	0.1308	b1	1.0557	0.122	b1	2.1265	0.237	b1	1.3839	0.1365
a2	0.7437	0.0717	a2	0.3113	0.0514	a2	0.1857	0.0406	a2	0.3289	0.0573
b2	1.437	0.1644	b2	1.6682	0.1929	b2	2.2223	0.2432	b2	2.0187	0.1639
a3	0.0495	0.0236	a3	0.214	0.1012	a3	0.0956	0.0405	a3	0.2143	0.0637
b3	2.1106	0.5693	b3	1.1461	0.6002	b3	1.4934	0.5177	b3	2.4114	0.2316
a4	0.0303	0.0154	a4	N/A	N/A	a4	N/A	N/A	a4	0.0445	0.0129
b4	2.4905	0.5521	b4	N/A	NA	b4	N/A	NA	b4	2.521	0.232
a5	0.1285	0.0102	a5	0.2794	0.0388	a5	0.2571	0.0387	a5	0.1201	0.0226
b5	1.0753	0.1378	b5	1.0166	0.1798	b5	1.1526	0.194	b5	1.7735	0.1733
Deerbrush			Snowbrush			*Ribes*					
Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error			
a1	0.5758	0.0118	a1	0.2517	0.0284	a1	0.1774	0.0199			
b1	2.5491	0.1528	b1	1.0534	0.1369	b1	1.6725	0.0937			
a2	0.0625	0.0159	a2	0.3041	0.0291	a2	0.1253	0.0242			
b2	1.9903	0.1991	b2	1.2848	0.1074	b2	2.3357	0.1452			
a3	0.0149	0.0062	a3	0.1064	0.0309	a3	0.0067	0.0035			
b3	0.785	0.441	b3	1.0362	0.2623	b3	2.3289	0.2462			
a4	N/A	N/A	a4	0.0201	0.0082	a4	0.0237	0.0044			
b4	N/A	N/A	b4	0.9999	0.3025	b4	2.4044	0.1576			
a5	0.0247	0.0047	a5	0.2152	0.033	a5	0.0001	2.20E−09			
b5	2.4016	0.1432	b5	1.1546	0.1453	b5	9.9887	1.1533			

the 1-h, 10-h, 100-h + 1000-h, and leaf biomass components for MLR (Table 8). When comparing RMSE for the individual component SUR and MLR methods, RMSE for the 1-h component was 23.1% lower, RMSE for the 10-h component was 23.9% lower, and RMSE for the leaf component was 46.5% lower for serviceberry using MLR. There was a noticeable difference with the combined 100-h + 1000-h component for mountain whitethorn using MLR and the individual component SUR method. The RMSE for the mountain whitethorn 100-h component using SUR was 2.3594 kg and 0.4857 kg for the 1000-h component. The combined 100-h + 1000-h component RMSE using MLR was 0.885. Although RRMSE for this component using MLR was 170.98%, the overall reduction in RMSE demonstrates that the MLR method is accounting for the variation present within larger biomass components more efficiently than the SUR model is. MLR mostly resulted in low RMSE values and provided estimates that were unbiased, except when a component (1000-h) was omitted. This is because MLR ensures that the predicted component proportions all add to 1 (Poudel and Temesgen, 2015). Given these results, using the 100-h + 1000-h component to obtain proportions of component biomass would be the preferred approach to estimate total shrub biomass for MLR, given similar biomass components and shrub species.

4. Discussion

4.1. Equation development

Large shrubs measured in the field have a major influence on the accuracy (higher RMSPE) of the power model for mountain whitethorn, snowbrush, and serviceberry. Due to the nature in which shrubs sprout, branch, and spread, it is not unusual to encounter difficulties in predicting shrub biomass (McGinnis et al., 2010). Since the size and shapes of shrubs can vary greatly, regressions of fuel component versus size, age, and other characteristics may not be strong (Martin et al., 1981). Deerbrush and *Ribes* spp. can grow tall and wide, and these types of allometric relationships can prove to be challenging when attempting to predict total aboveground shrub biomass. McGinnis et al. (2010) found that when crown diameter reached a maximum of 318 cm for *Ribes* spp. total biomass, mean squared error was larger (0.731 cm) when compared to shrubs that possessed smaller crown diameters within their study. However, this variability is not exclusive to shrub biomass only. To a certain extent, many conifer species also possess complex allometric relationships when they are young and obtain somewhat uniform stems and radially symmetric branches by the time they reach middle age. Some of these conifers may return to a complex form resulting from fire, pathogens, or other environmental factors later in life (McGinnis et al., 2010). Wang (2006) also found that equations for stem biomass of Manchurian walnut (*Juglans mandshurica*) were poor because the growth form of the walnut frequently had no single, distinct stem. The choice of the appropriate allometric equation in which to estimate biomass involves a tradeoff involving precision, simplicity, and practical application (Wang, 2006).

The most instability (high RRMSPE) is associated with the larger diameter fuel components (e.g. the 100-h biomass component). The limited sample size in the 1000 h fuels might have contributed to high RRMSPE. The inflated RRMSPE for the 100-h biomass components and the low values of RRMSPE for the 1-h and leaf biomass components obtained in this study are consistent with past shrub biomass studies in which foliage, live, and dead biomass were compiled into the same size classes (Murray and Jacobson, 1982; Sağlam et al., 2008; Grigal and Ohmann, 1977). The estimation of threetip sagebrush biomass using height and stem diameter similarly resulted in high adjusted R^2 values for leaves (0.89), leaves and live twigs <0.6 cm (0.86), and live twigs <0.6 cm (0.95) using varying forms of the power equation and log-log linear regression while separating live wood from dead wood by size class (Murray and Jacobson, 1982). Similarly, live twigs and branches that were greater than 2.54 cm (100-h biomass component) in diameter experienced a sharp decline in adjusted R^2 (0.09). The Murray and Jacobson (1982) study provides evidence that suggests that as biomass component size increases for live branches, the proportion of variation within component class is poorly accounted for by predictor variables, such as stem diameter and height. The results obtained by Murray and Jacobson (1982) for model accuracy regarding live twigs and branches greater than 2.54 cm in diameter

Table 7
Number of observations (n) by biomass component, parameter estimates, RMSE, and RMSE% for individual biomass component by species resulting from multinomial logistic regression (MLR). a and b are regression coefficients and RMSE = root mean square error (kg).

Serviceberry	n	a	b	RMSE	RMSE%
1-h	28	Base	Base	0.08	22.16
10-h	24	−0.1452	0.3908	0.14	25.71
100-h	8	−0.9047	−0.4581	0.09	129.18
1000-h	1	−9.2315	3.4183	0.11	400.00
Leaf	28	−1.1923	−0.1144	0.04	42.37
Greenleaf manzanita	n	a	b	RMSE	RMSE%
1-h	31	Base	Base	0.07	32.96
10-h	30	0.2156	0.2983	0.16	38.37
100-h	16	−0.1000	0.0961	0.27	127.92
1000-h	N/A	N/A	N/A	N/A	N/A
Leaf	31	0.2427	0.0119	0.14	54.43
Bush chinkapin	n	a	b	RMSE	RMSE%
1-h	19	Base	Base	0.07	28.70
10-h	18	0.2569	0.0745	0.09	28.15
100-h	6	−1.5993	0.3529	0.12	137.97
1000-h	N/A	N/A	N/A	N/A	N/A
Leaf	19	0.6769	−0.3380	0.09	39.06
Mountain whitethorn	n	a	b	RMSE	RMSE%
1-h	25	Base	Base	0.73	149.33
10-h	21	−0.6882	0.3167	0.11	19.30
100-h	10	−0.0840	−0.0370	0.78	191.09
1000-h	3	−2.7671	0.4749	0.12	113.07
Leaf	25	−0.5630	−0.1780	0.11	63.26
Deerbrush	n	a	b	RMSE	RMSE%
1-h	21	Base	Base	0.03	12.87
10-h	20	−0.1212	−0.1328	0.03	23.68
100-h	6	−1.5335	−0.5450	0.02	136.20
1000-h	N/A	N/A	N/A	N/A	N/A
Leaf	21	−0.8835	−0.0422	0.02	21.79
Snowbrush	n	a	b	RMSE	RMSE%
1-h	26	Base	Base	0.08	23.09
10-h	25	−0.0065	0.1434	0.09	17.75
100-h	16	−0.9644	0.0279	0.13	92.19
1000-h	2	−3.4105	0.2861	0.06	251.78
Leaf	26	−0.1750	0.0357	0.16	47.55
Ribes	n	a	b	RMSE	RMSE%
1-h	26	Base	Base	0.07	27.99
10-h	22	−0.4384	0.2647	0.14	39.49
100-h	10	−1.3452	−0.2585	0.06	168.49
1000-h	2	−8.5853	1.9687	0.03	63.00
Leaf	26	−1.1564	0.2774	0.12	69.81

are similar to the findings of this study, with the choice of the predictor variable differing for this research.

4.2. Assessment of fitting strategies

A total of 35 allometric equations were derived using nonlinear weighted least squares regression for seven species of shrubs. An allometric model was used to fit aboveground biomass components (total, 1-h, 10-h, 100-h, and leaf (kg)) as a function of crown area. The model performed well within all components for deerbrush, *Ribes* spp. (currant and gooseberry), and bush chinkapin. The power model performed best within the 1-h and leaf biomass components and was unstable for all species within the 100-h component. Large shrub observations obtained in the field affected model accuracy, especially for mountain whitethorn, snowbrush, and serviceberry. Model fits were unbiased for most species and biomass components.

Variability was present within this dataset and evident by large RMSPE and RRMSPE values resulting from model fits for some of the species and their respected components. Due to this variability, total biomass component estimation for mountain whitethorn and snowbrush resulted in large values of predicted error, however, these estimates will be unbiased using weighted nonlinear least squares regression. Previous research has examined alternative methods to estimating woody plant biomass, including Parresol (2001), who found that nonlinear seemingly unrelated regression resulted in lower variance because it considers contemporaneous correlations.

Seemingly unrelated regression was applied to create combined wood and individual component models for total biomass estimation. The individual component SUR model performed worse when compared to the combined wood SUR model. The grouping of all wood biomass classes resulted in lower values of RMSE for all species when compared to the individual component model. The individual component model would be useful to researchers and landowners who specifically need individual component biomass parameter estimates.

Using the combined wood SUR model is an efficient approach to estimate biomass since observed or actual total biomass information may not be available. Seemingly unrelated regression allows for the inclusion of dependencies among the error terms of the component biomass equation and is a commonly used method of component biomass estimation (Parresol, 1999). This method may also be applied to multinomial log-linear regression (MLR), which is used to obtain predicted proportions of biomass in each component (Poudel and Temesgen, 2015). To obtain predicted proportions of biomass components, the modeler would first use the combined wood SUR coefficients to obtain predicted values

Table 8
Number of observations (n) by component, parameter estimates, RMSE, and RMSE%, resulting from combining 100-h and 1000-h components and using multinomial logistic regression (MLR). a and b are regression coefficients and RMSE = root mean square error (kg).

Serviceberry	n	a	b	RMSE	RMSE%
1-h	28	Base	Base	0.07	21.38
10-h	24	−0.1242	0.3779	0.14	25.03
100-h + 1000-h	9	−1.6919	0.2791	0.14	137.25
Leaf	28	−1.1982	−0.1104	0.04	41.35
Greenleaf manzanita	n	a	b	RMSE	RMSE%
1-h	31	Base	Base	0.07	32.96
10-h	30	0.2155	0.2983	0.16	38.37
100-h + 1000-h	16	−0.1000	0.0960	0.27	127.92
Leaf	31	0.2428	0.0118	0.14	54.43
Bush chinkapin	n	a	b	RMSE	RMSE%
1-h	19	Base	Base	0.07	28.70
10-h	18	0.2570	0.0744	0.09	28.15
100-h + 1000-h	6	−1.5997	0.3531	0.12	137.97
Leaf	19	0.6762	−0.3378	0.09	39.06
Mountain whitethorn	n	a	b	RMSE	RMSE%
1-h	25	Base	Base	0.72	148.31
10-h	21	−0.6811	0.3140	0.11	18.68
100-h + 1000-h	13	−0.0978	0.0615	0.89	170.98
Leaf	25	−0.5668	−0.1765	0.11	62.59
Deerbrush	n	a	b	RMSE	RMSE%
1-h	21	Base	Base	0.03	12.87
10-h	20	−0.1213	−0.1328	0.03	23.68
100-h + 1000-h	6	−1.5331	−0.5451	0.02	136.20
Leaf	21	0.8835	−0.0422	0.02	21.79
Snowbrush	n	a	b	RMSE	RMSE%
1-h	26	Base	Base	0.08	23.06
10-h	25	−0.0062	0.1433	0.09	17.75
100-h + 1000-h	18	−0.9031	0.0666	0.14	78.87
Leaf	26	−0.1749	0.0357	0.16	47.55
Ribes	n	a	b	RMSE	RMSE%
1-h	26	Base	Base	0.06	23.96
10-h	22	−0.4267	0.2605	0.15	44.88
100-h + 1000-h	12	−2.0871	0.3398	0.05	48.28
Leaf	26	−1.1442	0.2731	0.11	64.40

Table 9
Number of observations (n) by components, parameter estimates, RMSE, RMSE%, bias, and bias% resulting from combining the 10-h and 100-h components and dropping 1000-h component using multinomial logistic regression (MLR). a and b are regression coefficients and RMSE = root mean square error (kg).

Serviceberry	n	a	b	RMSE	RMSE%	Bias	Bias%
1-h	28	Base	Base	0.08	22.28	−0.08	−2.38
10-h + 100-h	32	0.1518	0.2834	0.16	24.63	−0.02	−2.87
Leaf	28	−1.1955	−0.1124	0.04	41.58	0.00	−2.16
Greenleaf manzanita	n	a	b	RMSE	RMSE%	Bias	Bias%
1-h	31	Base	Base	0.07	32.96	0.00	0.00
10-h + 100-h	46	0.7548	0.2280	0.19	30.38	0.00	0.00
Leaf	31	0.2428	0.0118	0.14	54.43	0.00	0.00
Bush chinkapin	n	a	b	RMSE	RMSE%	Bias	Bias%
1-h	19	Base	Base	0.07	28.57	0.00	0.00
10-h + 100-h	24	0.3876	0.1313	0.09	22.83	0.00	0.00
Leaf	19	0.6758	−0.3374	0.09	39.10	0.00	0.00
Mountain whitethorn	n	a	b	RMSE	RMSE%	Bias	Bias%
1-h	25	Base	Base	0.78	160.11	−0.03	−6.56
10-h + 100-h	31	0.2905	0.1544	0.58	59.43	−0.07	−6.77
Leaf	25	−0.5807	−0.1709	0.13	74.56	−0.01	−6.56
Deerbrush	n	a	b	RMSE	RMSE%	Bias	Bias%
1-h	21	Base	Base	0.03	12.57	0.00	0.00
10-h + 100-h	26	0.0536	−0.1659	0.04	27.76	0.00	0.00
Leaf	21	−0.8846	−0.0419	0.02	21.56	0.00	0.00
Snowbrush	n	a	b	RMSE	RMSE%	Bias	Bias%
1-h	26	Base	Base	0.08	23.34	−0.01	−1.85
10-h + 100-h	41	0.3112	0.1180	0.15	22.36	−0.01	−1.92
Leaf	26	−0.1750	0.0357	0.16	48.47	−0.01	−1.89
Ribes	n	a	b	RMSE	RMSE%	Bias	Bias%
1-h	26	Base	Base	0.07	27.24	−0.01	−4.89
10-h + 100-h	32	−0.1669	0.2066	0.22	59.21	−0.02	−6.15
Leaf	26	−1.1517	0.2757	0.11	60.55	−0.01	−6.56

of leaf and wood, since the predicted value of Total $\widehat{\text{biomass}} = \widehat{\text{wood}} + \widehat{\text{leaf}}$. One can use either the predicted or the observed (if available) value of total biomass to obtain the proportion of total shrub biomass by fuel class component (Poudel and Temesgen, 2015). Combining the wood observations helped decrease error somewhat, however, inference concerning individual fuel classes is also lost in this process.

Multinomial log-linear regression was also employed to estimate the component biomass for each shrub. The MLR method produced RMSE values that were lower than both the combined wood and individual component SUR methods. Combining the 100-h and 1000-h biomass components using MLR resulted in low values of RMSE across all species and biomass components. MLR provided estimates that were unbiased, except when a component (1000-h) was omitted from the model fit. This is a flexible approach to obtaining proportional component biomass because fuel classes containing small numbers of observations may be combined with another fuel class that has similar characteristics.

The results obtained by using the SUR and MLR approaches applied within this study have also been realized in other studies, including the estimation of aboveground biomass for Douglas-fir and lodgepole pine (*Pinus contorta* var. *latifolia* Engelm.) for different regions in Oregon (Poudel and Temesgen, 2015). That study found that the system of equations fitted using the SUR method were superior to analytical methods based on existing equations, in terms of bias and RMSE (Poudel and Temesgen, 2015). The predicted proportion method involving multinomial log-linear regression also produced smaller values of RMSE when compared to the SUR methods that were applied (Poudel and Temesgen, 2015). Other studies have looked to different regression methods to reduce bias and RMSPE. For example, Eskelson et al. (2011) used beta regression to estimate percent shrub cover, which produced smaller mean squared prediction error and absolute bias when compared to ordinary least squares regression models used in their study. Furthermore, Poudel and Temesgen (2016) found that even though measures of accuracy differed between species in western hemlock (*Tsuga heterophylla* (Raf.) Sarg.) and red alder (*Alnus rubra* Bong.) biomass estimation, lower values of RMSE were produced for multinomial log-linear regression when compared to the SUR, beta, and Dirichlet regression methods. The same methods (SUR, MLR) applied within this study yielded similar outcomes when total aboveground shrub biomass was considered.

A potential limitation involved with this study was the presence of excess zeros mostly found within the 100-h and 1000-h biomass components. Values of zero were prevalent within these components because as component size increases, there were fewer shrubs observed in the field that possessed such diameters. This led to difficulty in model fitting for some species (bush chinkapin and deerbrush). An option to address this limitation may be to use a model that can specifically address the issue of excessive zero values in data sets. This option will be examined in a follow-up study.

A recommendation that may help to improve future results for similar studies involves how biomass was grouped into components. While results were accurate for the 1-h and leaf components, results were not nearly as accurate for the 10-h and 100-h classes. Studies that separated live and dead branches were also able to obtain high (0.90+) R^2 values for total shrub biomass estimation (Murray and Jacobson, 1982). Separating live and dead twigs and branches from one another may provide more accurate estimates within the components used in this study, but this would be a time consuming and costly endeavor. Combining the 100-h and 1000-h biomass components was also beneficial in obtaining low values of RMSE and unbiased estimates by implementing MLR.

5. Conclusion

The equations resulting from this research are applicable to areas within northeastern California where similar climate, soils, and vegetation associations may be found in relation to the study area for which this research occurred. Each of the methods examined within this study have their benefits, but the decision of which one to utilize will ultimately depend upon the researcher's or landowner's objectives. Applying such methods in other parts of northeastern California with similar shrub species would help to validate model accuracy. The allometric equations will be useful to forest modelers interested in the assessment of total and proportional component biomass for carbon accounting and for fire modelers concerned with forest fuel accumulation and wildfire prevention.

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